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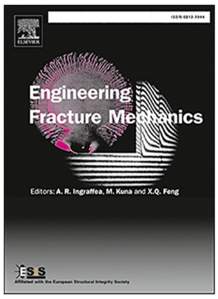

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On the Stress Intensity Factor of cracks emanating from circular and elliptical holes in orthotropic plates

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Abstract

Stress Intensity Factors (SIFs) for cracks emanating from circular holes in two-dimensional orthotropic bodies were numerically computed taking into account the effect of geometry and orthotropy. A semi-analytical expression for the correction factor was found fitting the numerical data. Finally, it was demonstrated how the same expression can be used to calculate the SIF for cracks emanating from elliptical holes once appropriate changes of variables are made.

Keywords: Open Hole Specimen, Stress Intensity Factor (SIF), Finite Element Analysis (FEA), Ortotropic Rescaling Technique

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Nomenclature

a, crack length

c, ellipse aspect ratio

i, j, k, l, indexes

s_{ij}, element of the compliance matrix

w, semi-width of the holed plate

x_1, x_2, natural axes of the body

$\acute{x_2}$, rescaled coordinate

E_1, Young's modulus in x_1 direction

E_2, Young's modulus in x_2 direction

$\acute{E}$, equivalent modulus

$\mathcal{G}_I$, energy release rate in mode I

G_{12}, in-plane shear modulus

$\mathcal{K}_I$, stress intensity factor in mode I

L, semi-length of the holed plate

P_{ijkl}, fitting parameter

U, Airy stress function

R, Radius of the hole

α, normalised crack length

ζ, normalised length

λ, dimensionless elastic constant

$\acute{\lambda}$, rescaled dimensionless elastic constant

ν_{ij}, minor or major Poisson's ratio

ξ, dimensionless variable function of α

ρ, dimensionless elastic constant

σ^{∞}, remote tensile stress

ϕ, correction factor (circular hole)

ψ, correction factor (elliptical hole)

ω, normalised width

ERR, Energy Release Rate

FEA, Finite Element Analysis

FEM, Finite Element Method

FFMs, Finite Fracture Mechanics

SIF, Stress Intensity Factor

1. Introduction

Holes are one of the most common structural features found in aeronautic structures because they are used in mechanically fastened joints and are also used to enable the connection of different systems using cables. The strength of notched composite plates in fibre-reinforced composite laminates can be promptly estimated using both advanced numerical progressive damage models [1] and simple analytical methods [2, 3] based on Finite Fracture Mechanics (FFMs). FFMs requires solving a system of two equations that represent coupled stress and energy based criteria. Clearly, an expression of the Energy Release Rate (ERR), or equivalently, of the Stress Intensity Factor (SIF) needs to be available. If for quasi-isotropic laminates the SIF for isotropic materials can be used [2], for general orthotropic laminates [4] an expression for the SIF is not available.

Here, this problem will be solved providing an expression for the SIF of rectilinear cracks emanating from circular holes in plates loaded in mode I by a remote tensile stress. Moreover, it will also be shown that with an appropriate change of variables the same expression will also provide the SIF for cracks emanating from elliptical holes.

2. Circular holes

Let $2w$, $2L$, and $2R$ be respectively the width, the length, and the diameter of the notched plate depicted in Fig. 1. Let a be the length of two rectilinear cracks originating from the notch that propagates along the x_1 direction. Let x_1 and x_2 be the two natural axes of the orthotropic body. The plate is loaded with a remote tensile stresses σ^∞, and the two cracks are loaded in mode I. Plane stress states in the plane $x_1 - x_2$ is assumed.

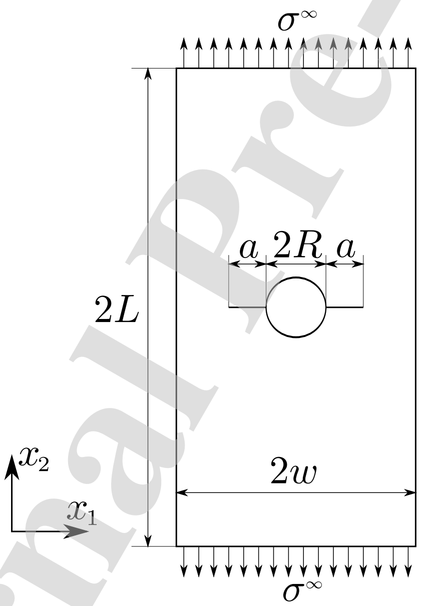

Figure 1: Cracks emanating from a circular hole in a finite width plate.

The SIF takes the expression:

$$\mathcal{K}_I = \sqrt{R}\sigma^\infty\phi \tag{1}$$

where ϕ is a correction factor that depends on the shape of the plate and on the orthotropy of the material. Formally, the correction factor takes the form:

$$\phi = \phi\left(\alpha, \rho, \lambda, \omega, \zeta\right) \tag{2}$$

where α is the normalised crack length, $\alpha = a/R$; ω is the normalised width, $\omega = w/R$; ζ is the normalised length $\zeta = L/R$; and λ and ρ are two materials parameters defined as [5]:

$$\lambda = \frac{s_{11}}{s_{22}}, \quad \rho = \frac{2s_{12} + s_{66}}{2\sqrt{s_{11}s_{22}}} \tag{3}$$

where s_{ij} are the elements of the compliance matrix that can be expressed in terms of the engineering constants as:

$$s_{11} = \frac{1}{E_1}, \quad s_{22} = \frac{1}{E_2}, \quad s_{66} = \frac{1}{G_{12}}, \quad s_{12} = -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2} \tag{4}$$

For very long plates ($\zeta > 4\omega$) the stress intensity factor does not depend on the length of the specimen and therefore the dependence on ζ can be eliminated from Equation (2). Moreover, it is convenient to define a new dimensionless variable, ξ, as:

$$\xi = \frac{\alpha}{\omega - 1} \tag{5}$$

and, since α ranges between $[0, \omega - 1]$, $\xi \in [0, 1]$. Therefore Equation (2) can be rewritten as:

$$\phi = \phi\left(\xi, \rho, \lambda, \omega\right) \tag{6}$$

An expression for Equation (6) can be deduced by finding an appropriate fitting function for data generated through Finite Element Analyses (FEA) as shown in [6, 7]. For any combination of input variables, $(\xi^*, \rho^*, \lambda^*, \omega^*)$ the corresponding values of the ERR, $\mathcal{G}_I^*$, can be computed, and since the SIF and ERR are related, $\mathcal{G}_I = \mathcal{K}_I^2/\acute{E}$, being $\acute{E} = \left[s_{11}s_{22}\left(1+\rho\right)/2\right]^{1/2}\lambda^{-1/4}$, the equivalent modulus, the corresponding value of the correction factor is readily found as $\phi\left(\xi^*, \rho^*, \lambda^*, \omega^*\right) = \sqrt{\mathcal{G}_I^* \acute{E}^*/R^*}/\sigma^*$ where R^* and σ^* are the radius and remote stress used in the FEM model.

Values for the correction factor were computed for $\lambda \in [0.05, 16]$, $\rho \in [0, 10]$, $\omega \in [2, 10]$ and $\xi \in [0.02, 0.9]$. Noticing that $\phi\left(0, \rho, \lambda, \omega\right) = 0$ (there is no crack when $\xi = 0$) and $\phi\left(1, \rho, \lambda, \omega\right) = \infty$ (when $\xi = 1$ there is no ligament and the crack has reached its maximum extension), it is appropriate to chose the following expression for ϕ:

$$\phi = \left(\tan\frac{\pi\xi}{2}\right)^{\frac{1}{4}} \sum_{i,j,k,l} P_{ijkl}\xi^i\rho^j\lambda^{-\frac{1}{4}k}\omega^l \tag{7}$$

where P_{ijkl} are the parameters that are computed using the Levenberg-Marquardt algorithm (Table 1). As shown in Figure 2, the fitting function matches very well the numerical data, and ensures a mean error of about 2% in the selected domain.

Table 1: Parameters of Equation (7), P_{ijkl}.

(j,k,l)	i=0	i=1	i=2	i=3
(0,0,0)	3.66E+00	1.39E+01	-4.18E+01	3.11E+01
(0,0,1)	1.81E-01	-8.42E+00	2.05E+01	-1.22E+01
(0,0,2)	-1.91E-02	1.25E+00	-2.87E+00	1.73E+00
(0,0,3)	6.88E-04	-5.40E-02	1.22E-01	-7.33E-02
(0,1,0)	-8.54E-01	-2.11E+00	2.63E+01	-2.31E+01
(0,1,1)	-1.23E-01	5.16E+00	-1.87E+01	1.37E+01
(0,1,2)	3.95E-02	-8.70E-01	2.81E+00	-1.99E+00
(0,1,3)	-2.14E-03	4.10E-02	-1.28E-01	8.91E-02
(1,0,0)	3.37E-01	-6.13E+00	1.46E+01	-9.21E+00
(1,0,1)	-1.78E-01	2.76E+00	-6.46E+00	3.89E+00
(1,0,2)	2.55E-02	-3.77E-01	8.66E-01	-5.19E-01
(1,0,3)	-1.15E-03	1.66E-02	-3.76E-02	2.24E-02
(1,1,0)	-1.65E-01	3.64E+00	-1.11E+01	7.70E+00
(1,1,1)	1.29E-01	-2.19E+00	5.95E+00	-3.92E+00
(1,1,2)	-2.10E-02	3.27E-01	-8.61E-01	5.57E-01
(1,1,3)	1.00E-03	-1.50E-02	3.88E-02	-2.49E-02
(2,0,0)	-2.03E-02	3.95E-01	-9.58E-01	6.06E-01
(2,0,1)	1.09E-02	-1.79E-01	4.26E-01	-2.59E-01
(2,0,2)	-1.59E-03	2.45E-02	-5.72E-02	3.45E-02
(2,0,3)	7.22E-05	-1.08E-03	2.48E-03	-1.49E-03
(2,1,0)	1.28E-02	-2.62E-01	7.71E-01	-5.27E-01
(2,1,1)	-8.77E-03	1.51E-01	-4.08E-01	2.67E-01
(2,1,2)	1.40E-03	-2.25E-02	5.89E-02	-3.79E-02
(2,1,3)	-6.70E-05	1.03E-03	-2.66E-03	1.69E-03

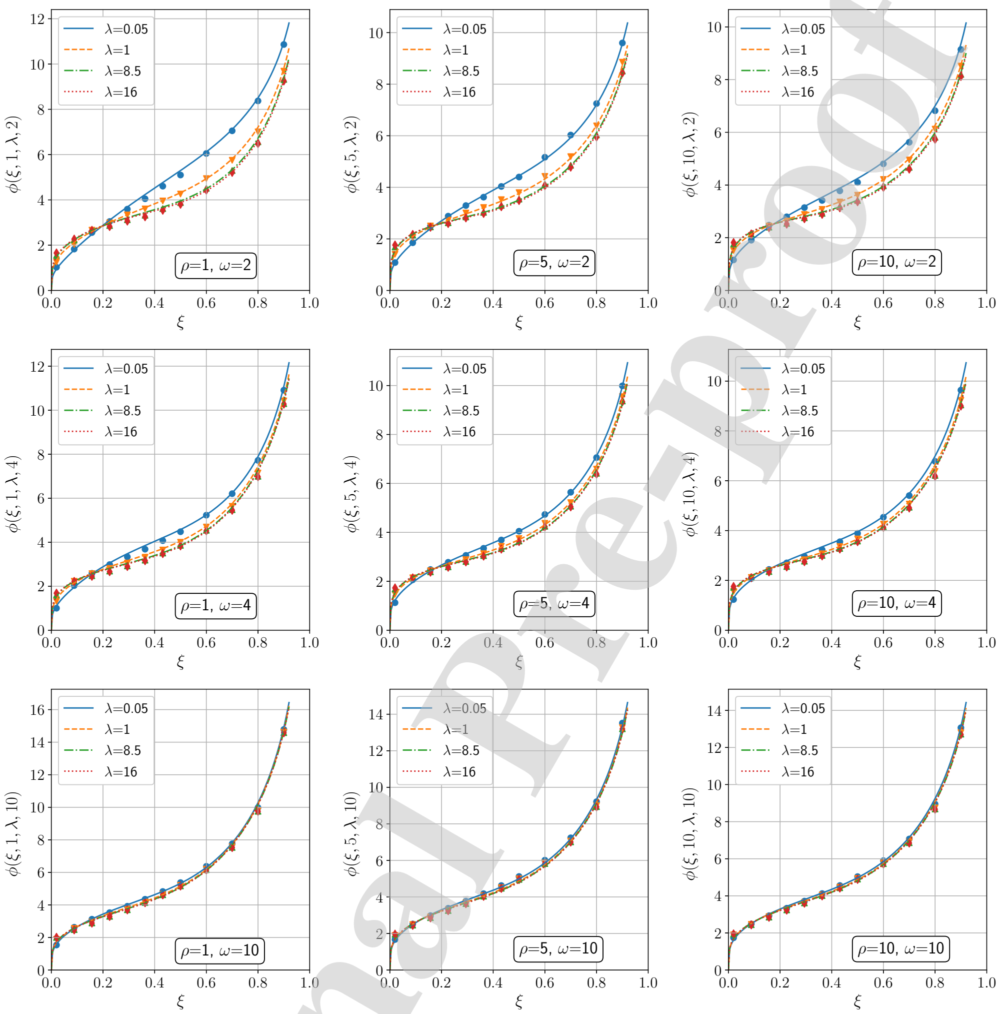

Figure 2: Fitting function (lines) and FEM generated data (points) for different input parameters.

3. Elliptical holes

Consider now the orthotropic plate with an elliptical notch of Figure 3(a). Let the two axes of the ellipse be R and cR long ($c > 0$), where c is the ellipse aspect ratio or its inverse if $c > 1$ or $0 < c < 1$, respectively.

Supposing that the length of the specimen is large enough not to influence the calibration of the

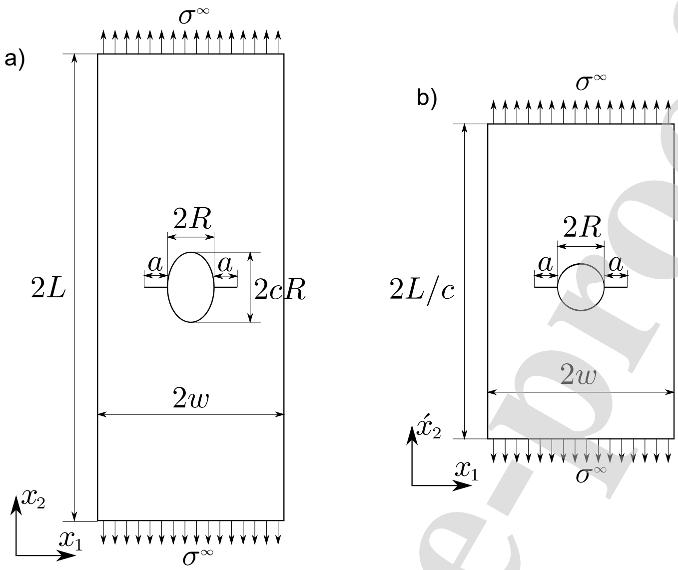

Figure 3: Elliptical-holed plate and equivalent circular-holed plate.

correction factor ψ, the SIF reads:

$$\mathcal{K}_I = \sqrt{R}\sigma^\infty \psi\left(\alpha, \rho, \lambda, \omega, c\right) \tag{8}$$

where now the correction factor depends also on c. The calculation of ψ can be done following the procedure explained in the previous section. However, in this case, it possible to derive an expression for ψ without performing any additional numerical simulation. In fact, at equilibrium, the stress field in the body reads:

$$\sigma_1 = \frac{\partial^2 U}{\partial x_2^2}, \quad \sigma_2 = \frac{\partial^2 U}{\partial x_1^2}, \quad \tau_{12} = \frac{\partial^2 U}{\partial x_1 \partial x_2} \tag{9}$$

where U is the Airy stress function, while the compatibility equation reads:

$$\frac{\partial^4 U}{\partial x_1^4} + 2\rho\sqrt{\lambda}\frac{\partial^4 U}{\partial x_1^2 \partial x_2^2} + \lambda\frac{\partial^4 U}{\partial x_2^4} = 0 \tag{10}$$

Rescaling the x_2-axis, posing $\acute{x}_2 = x_2/c$, causes a contraction ($c > 1$) or dilatation ($0 < c < 1$) of the plate along the x_2 direction. In the new coordinate system, x_1-$\acute{x}_2$, the plate has a circular hole, as shown in Figure 3(b). With this change of variables, the compatibility equation reads:

$$\frac{\partial^4 U}{\partial x_1^4} + 2\rho\frac{\sqrt{\lambda}}{c^2}\frac{\partial^4 U}{\partial x_1^2 \partial x_2^2} + \frac{\lambda}{c^4}\frac{\partial^4 U}{\partial x_2^4} = 0 \tag{11}$$

Using $\acute{\lambda} = \lambda/c^4$, and replacing in Equation (11), yields:

$$\frac{\partial^4 U}{\partial x_1^4} + 2\rho\sqrt{\acute{\lambda}}\frac{\partial^4 U}{\partial x_1^2 \partial x_2^2} + \acute{\lambda}\frac{\partial^4 U}{\partial x_2^4} = 0 \tag{12}$$

Comparing Equation (12) to Equation (10) reveals that the SIF for the elliptical-holed plate (Figure 3(a)) can be determined computing the SIF of the equivalent circular-holed plate (Figure 3(a)) where λ is replaced with $\acute{\lambda} = \lambda/c^4$. Therefore, the correction factor for the elliptical-holed plate reads[1]:

$$\psi(\alpha, \rho, \lambda, \omega, c) = \phi\left(\alpha, \rho, \frac{\lambda}{c^4}, \omega\right) \tag{13}$$

As a demonstrative example, the correction factor for elliptical-holed cracked plates was numerically computed for plates with $c = 0.5, 1.5, 2$; $\rho = 8$; $\lambda = 1$; and $\omega = 4, 6, 8$. The comparison between the numerically generated data, and the predictions obtained using the expression in Equation (13), is excellent as shown in Figure 4.

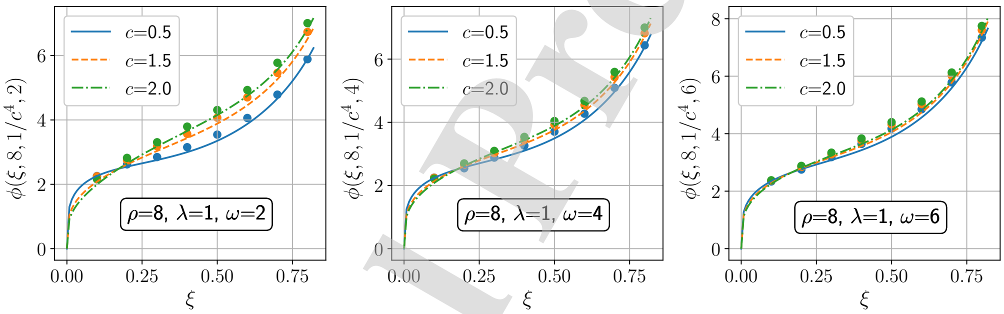

Figure 4: Elliptical-holed plate and equivalent circular-holed plate.

Conclusions

An expression for the correction factor of cracks emanating from circular holes in finite-width orthotropic plates was found. Numerical models were used to determine the value of the correction factor and an appropriate function was defined to fit the data. With an appropriate change of variables, it was demonstrated that the same expression can be used to compute the correction factor for cracks emanating from elliptical holes. The expression provided will be used to extend the range of applicability of the FFMs models to the case of general orthotropic laminates.

[1] Assuming the length of the equivalent specimen, $2L/c$, to be large.

Declarations of interest

None.

Data availability

Datasets related to this article can be found at http://dx.doi.org/10.17632/5468zb93fx.1, an open-source online data repository hosted at Mendeley Data.

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Highlights

- SIFs were computed for cracks emanating from circular holes in orthotropic plates.
- A semi-analytical expression for the correction factor was proposed.
- A change of variables yields the SIFs for cracks emanating from elliptical holes.

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: