### Notas de Geotermia e Energia Geotérmica

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Aulas de Mestrado Recursos Hidrogeológicos e Geoenergia

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> > (Versão 2021)

## The Heat Conduction Equation

### Introduction

$$q_z = -\frac{\Delta Q}{A \cdot \Delta t} = -K \frac{dT}{dz}$$

Fourier equation

$$\frac{\partial T}{\partial t} = \alpha \cdot \frac{\partial^2 T}{\partial z^2}$$

Heat conduction equation

$$T = T_0 + \frac{Q_0}{K} \cdot z - \frac{A'}{2K} \cdot z^2$$

### Part 1

# General aspects of thermal energy in the Earth

## Energy balance at the Earth's surface

- The solar energy that reaches the Earth per unit time is  $2 \times 10^{17}$  W ( $4 \times 10^2$  Wm<sup>-2</sup>)
- The energy from the Earth's interior that reaches the surface per unit time is  $4.4 \times 10^{13} \text{ W}$  (8.7 x  $10^{-2} \text{ Wm}^{-2}$ )
- The energy per unit time spent to desaccelerate Earth's rotation (tides) is about 10<sup>12</sup> W
- The amount of energy per unit time released in earthquakes is about 10<sup>11</sup> W.

## Energy balance at the Earth's surface

(normalized to mean HFD at the Earth's surface)

<ul> <li>Solar energy</li> </ul>	4000
<ul> <li>Mean HFD at the Earth's surface</li> </ul>	1
<ul> <li>Desacceleration by tides</li> </ul>	0,1
<ul> <li>Earthquakes</li> </ul>	0,01

### Geothermal energy manifestations

Direct evidences

- Volcanos
- Thermal springs
- Temperature measured in boreholes and mines

### Geothermal energy manifestations

Indirect evidences

- Plate tectonic motions
- Seismic activity
- Orogeny
- Earth's magnetic field
- Metamorphic processes

### Thermal history of the Earth

- It is not possible to reconstruct the Earth's thermal history
- However, plausible models can be constructed based on:
  - The general heat transmission theory
  - Measurements of the heat flux from the Earth's interior
  - Radioactive studies and meaurements in rocks
  - Study of the physical properties of rocks

### Thermal history of the Earth

The firt attempt to estimate the Earth's age was done by William Thomson (Lord Kelvin)

$$t = \frac{T_0^2}{\left(\frac{dT}{dz}\right)_{z=0}^2 \cdot \pi \cdot \alpha}$$

$$T_0 = 3871 \, ^{\circ}\text{C} \quad (= 7000 \, ^{\circ}\text{F})$$

$$(dT/dz)_{z=0} = 0,036 \, ^{\circ}\text{C m}^{-1}$$

$$\alpha = 1,4 \times 10^{-6} \, \text{m}^2 \, \text{s}^{-1}$$

$$t = 100 \, \text{Ma}$$

#### Heat sources inside the Earth

- Uranium, thorium and potassium radioactive decay
- Conversion of gravitational energy into thermal energy
- Heat resulting from thermodynamic processes involved in the Earth's formation

### Temperature evolution inside the Earth

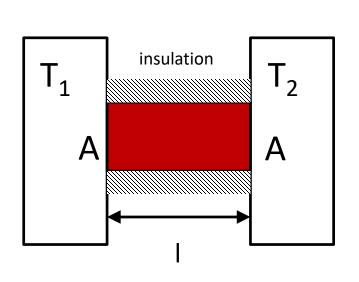
1. In a first phase, about 4.7 billion years, planetary formation and compression processes warmed the Earth's interior to temperatures of the order of 1000 °C

2. In a second phase, heat generation by radioactive processes started and the Earth's interior temperature continued to increase

### Temperature evolution inside the Earth

- 3. About 4 to 4.5 billion years, when the iron fusion temperature was reached, gravitational differention between the nucleous and the mantle started, which released gravitational energy of the order of 2 x 10<sup>30</sup> J
- 4. All these sources of thermal energy started fusion, reorganization, and differentiation inside the Earth, and the formation of a core, a mantle and a crust began

 Conduction – Thermal energy transfer as a kinetic energy transfer among the atoms and molecules of a given material



$$\Delta Q = K \cdot A \cdot \frac{T_2 - T_1}{L} \cdot \Delta t$$

$$q_z = -\frac{\Delta Q}{A \cdot \Delta t} = -K \frac{dT}{dz}$$

 Convection - Thermal energy transfer with motion of portions of matter with different densities resulting from temperature differences

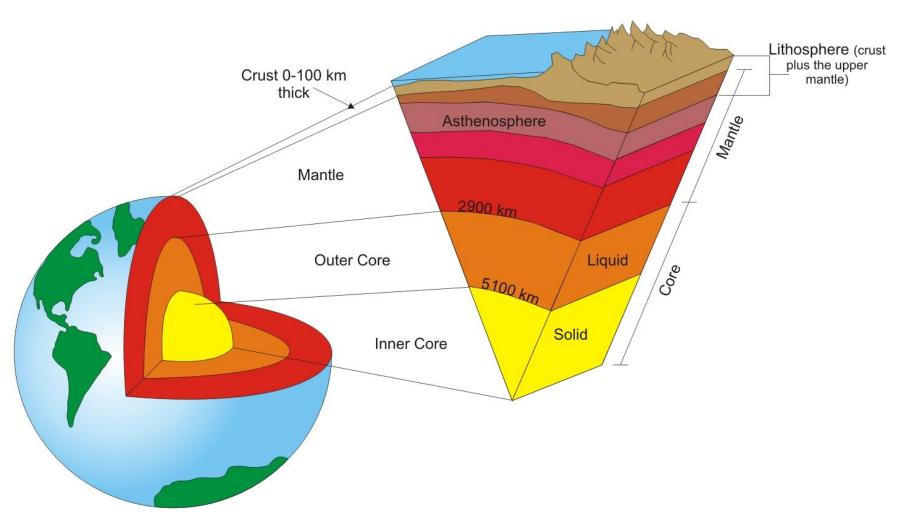
$$Ra = \frac{\alpha \cdot \rho \cdot g \cdot \Delta T}{K \cdot \eta} \cdot D^{3}$$

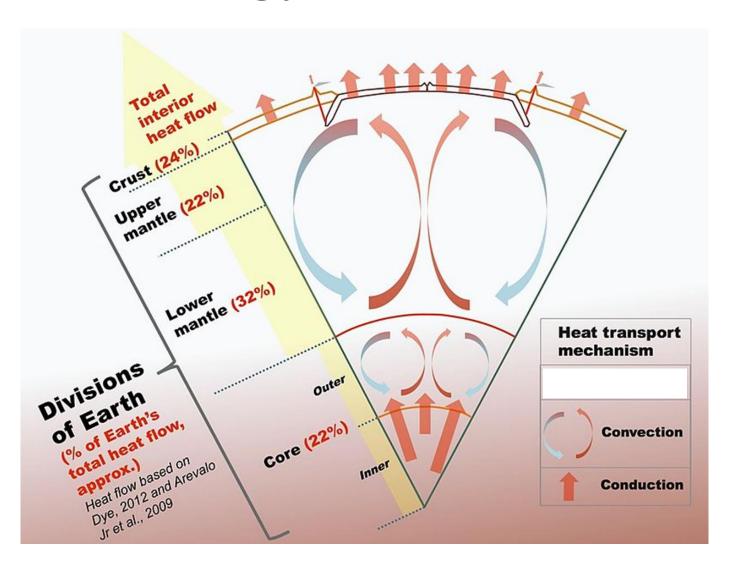
where Ra is the Rayleigh number,  $\alpha$  is the coefficient of thermal expansion,  $\rho$  is the density, g is the acceleration of gravity,  $\Delta T$  is the temperature difference, K is the thermal diffusivity,  $\eta$  is the viscosity and D³ is a measure of the volume involved in the convection. For Ra about  $10^3$  convection starts and at about  $10^5$  heat transfer is entirely by convection.

 Radiation – Thermal energy transfer as electromagnetic waves

$$R=\sigma\cdot T^{\scriptscriptstyle 4}$$

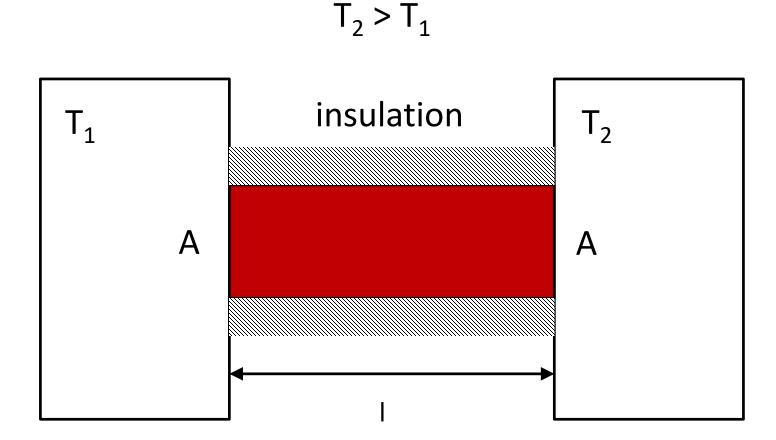
R represents the radiant energy per second emitted per unit area of the surface of the body at temperature T, and  $\sigma$  is the Stefan-Boltzman constant  $\sigma = 5,67 \times 10^{-8}$  Wm<sup>-2</sup>K<sup>-4</sup>

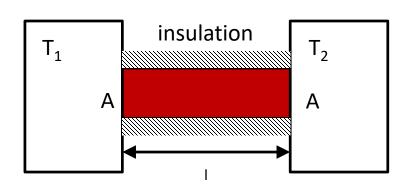




### Part 2

# About the conduction equation and some results





$$\Delta Q \propto A \cdot \frac{T_2 - T_1}{L} \cdot \Delta t$$

$$\Delta Q = K \cdot A \cdot \frac{T_2 - T_1}{L} \cdot \Delta t$$

$$q_z = -\frac{\Delta Q}{A \cdot \Delta t} = -K \frac{dT}{dz}$$

## Diversion about scalars, vectors and tensors

$$q_z = -\frac{\Delta Q}{A \cdot \Delta t} = -K \frac{dT}{dz}$$

 $q_z$  is a vector as well as the temperature gradient dT/dz. K is a tensor and the minus sign indicates the temperature gradient has an opposite direction of  $q_z$ . We are (almost) always assuming that  $q_z$  is vertical.

#### Diversion about units

$$\Delta Q = K \cdot A \cdot \frac{T_2 - T_1}{L} \cdot \Delta t$$

$$K = \frac{\Delta Q \cdot L}{\Delta t \cdot \Delta T \cdot A}$$

$$[K] = \frac{[\Delta Q] \cdot [L]}{[\Delta t] \cdot [\Delta T] \cdot [A]}$$

In SI, Q is in joules or J, T is in kelvin K (or °C), L is in meter, t is in second, A in m<sup>2</sup>. So, K should be in W/mK

$$[K] = \frac{J \cdot m}{s \cdot K \cdot m^2} = \frac{W}{mK} = W/mK = Wm^{-1}K^{-1}$$

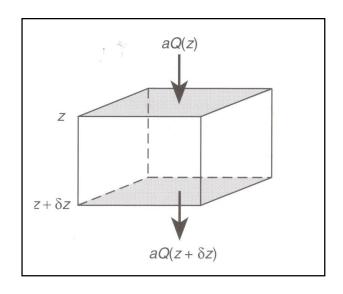
#### Diversion about units

What UNITS can do or not: the Mars Climate Orbiter case

(CNN) -- *September 30, 1999.* NASA lost a \$125 million Mars orbiter because one engineering team used metric units while another used English units for a key spacecraft operation, according to a review finding released Thursday. For that reason, information failed to transfer between the Mars Climate Orbiter spacecraft team at Lockheed Martin in Colorado and the mission navigation team in California.

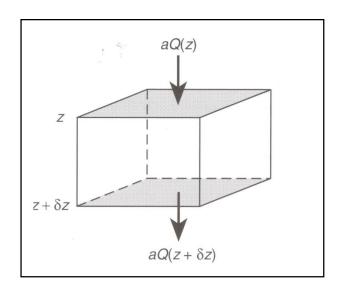
"People sometimes make errors," said Edward Weiler, NASA's Associate Administrator for Space Science in a written statement.

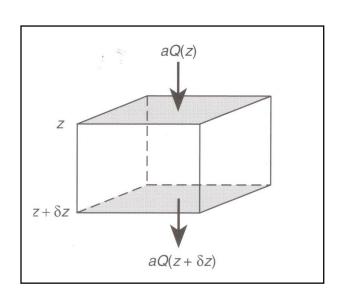
- C<sub>p</sub> is the specific heat
- ρ is the density
- A is the heat production per unit volume
- a is the area of the element of volume
- z is the depth
- Q is the amount of heat
- T is the temperature



What are the units of the quantities?

- C<sub>p</sub> is the specific heat (J/kg.°C)
- ρ is the density (kg/m³)
- A is the heat production per unit volume (W/m³)
- a is the area of the element of volume (m<sup>2</sup>)
- z is the depth (m)
- Q is the amount of heat (J)
- T is the temperature (K)





$$\mathbf{a} \cdot \mathbf{Q}(\mathbf{z}) - \mathbf{a} \cdot \mathbf{Q}(\mathbf{z} + \delta \mathbf{z}) = -\mathbf{a} \cdot \delta \mathbf{z} \cdot \frac{\partial \mathbf{Q}}{\partial \mathbf{z}}$$

$$A \cdot a \cdot \delta z$$

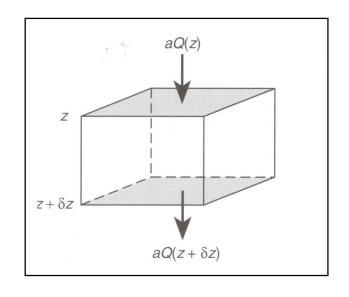
$$c_{P} \cdot a \cdot \delta z \cdot \rho \cdot \frac{\delta T}{\delta t}$$

$$c_{P} \cdot a \cdot \delta z \cdot \rho \cdot \frac{\delta T}{\delta t} = A \cdot a \cdot \delta z - a \cdot \delta z \cdot \frac{\partial Q}{\partial z}$$

$$c_{P} \cdot a \cdot \delta z \cdot \rho \cdot \frac{\delta T}{\delta t} = A \cdot a \cdot \delta z - a \cdot \delta z \cdot \frac{\partial Q}{\partial z}$$

Dividing by  $a\delta z$  and making  $\delta t$  go to 0, we obtain

$$c_{P} \cdot \rho \cdot \frac{\partial T}{\partial t} = A - \frac{\partial Q}{\partial z}$$



Since

$$Q_z = -K \frac{dT}{dz}$$

$$\frac{\partial T}{\partial t} = \frac{K}{c_p \cdot \rho} \cdot \frac{\partial^2 T}{\partial z^2} + \frac{A}{c_p \cdot \rho}$$

Heat conduction equation with heat production

$$\frac{\partial \mathbf{T}}{\partial \mathbf{t}} = \frac{\mathbf{K}}{\mathbf{c}_{p} \cdot \boldsymbol{\rho}} \cdot \frac{\partial^{2} \mathbf{T}}{\partial \mathbf{z}^{2}} + \frac{\mathbf{A}}{\mathbf{c}_{p} \cdot \boldsymbol{\rho}}$$

Heat conduction equation without heat production

$$\frac{\partial \mathbf{T}}{\partial \mathbf{t}} = \alpha \cdot \frac{\partial^2 T}{\partial z^2} \qquad \text{with} \qquad \alpha = \frac{\mathbf{K}}{\mathbf{c_p} \cdot \mathbf{\rho}}$$

Heat conduction equation with motion of material through the region where temperature changes with depth with velocity u<sub>7</sub>

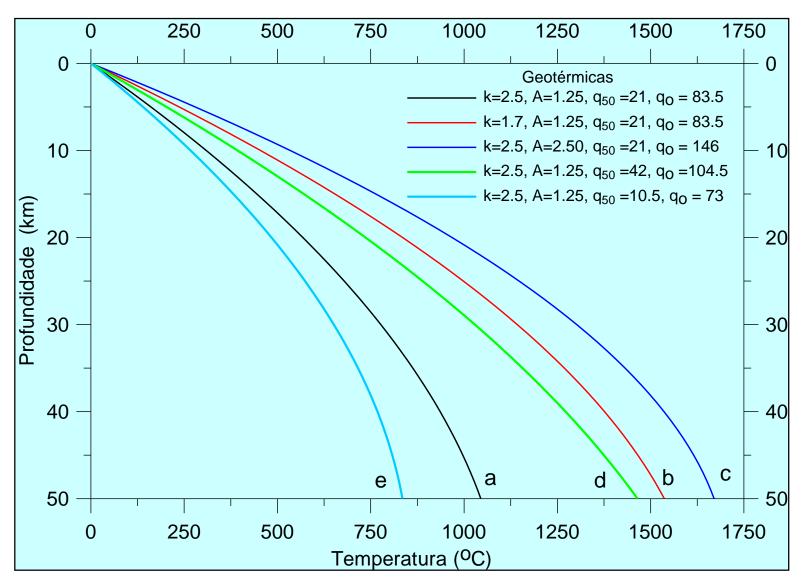
$$\frac{\partial \mathbf{T}}{\partial \mathbf{t}} = \alpha \cdot \frac{\partial^2 \mathbf{T}}{\partial z^2} + \frac{A}{\mathbf{c_p} \cdot \rho} - \mathbf{u_z} \cdot \frac{\partial \mathbf{T}}{\partial \mathbf{z}}$$

where  $u_z \cdot \frac{\partial T}{\partial z}$  is the advective transfer term

$$0 = \frac{K}{\rho c_{p}} \cdot \frac{\partial^{2} T}{\partial z^{2}} + \frac{A}{\rho c_{p}}$$

$$T = T_0$$
 for  $z = 0$   $Q = Q_0$  for  $z = 0$ 

$$T = T_0 + \frac{Q_0}{K} \cdot z - \frac{A}{2K} \cdot z^2$$

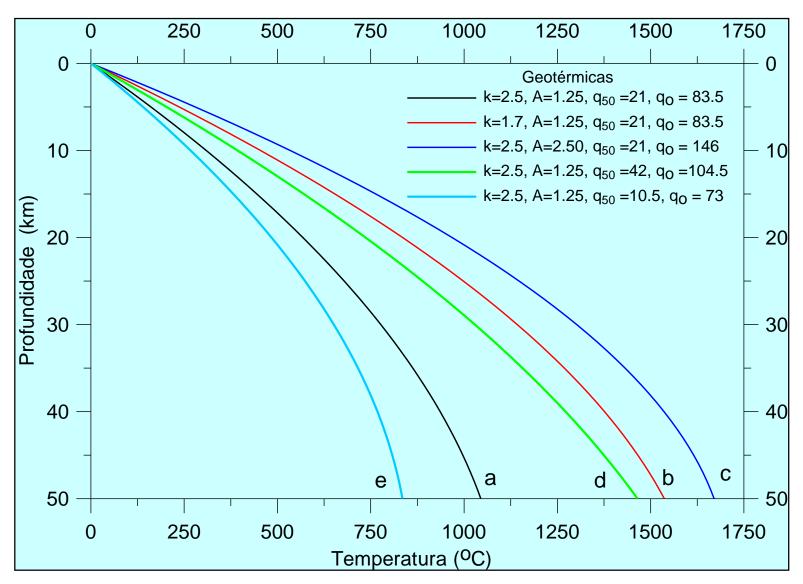


$$0 = \frac{K}{\rho c_{p}} \cdot \frac{\partial^{2} T}{\partial z^{2}} + \frac{A}{\rho c_{p}}$$

$$T = 0$$
 for  $z = 0$ 

$$T = 0$$
 for  $z = 0$   $Q = Q_d$  for  $z = d$ 

$$T = \frac{Q_d + Ad}{K} \cdot z - \frac{A}{2K} \cdot z^2$$



$$0 = \frac{K}{\rho c_{p}} \cdot \frac{\partial^{2} T}{\partial z^{2}} + \frac{A}{\rho c_{p}}$$

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 for  $z = 0$   
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$$T = T_0 + \frac{Q_0}{K} \cdot z - \frac{A}{2K} \cdot z^2$$

$$T = T_0 + \frac{Q_0}{K} \cdot z - \frac{A}{2K} \cdot z^2 \qquad T = \frac{Q_d + Ad}{K} \cdot z - \frac{A}{2K} \cdot z^2$$

What is the interesting thing about these two equations?

$$T = T_0 + \frac{Q_0}{K} \cdot z - \frac{A}{2K} \cdot z^2 \qquad T = \frac{Q_d + Ad}{K} \cdot z - \frac{A}{2K} \cdot z^2$$

Is that  $Q_0 = Q_d + Ad$ , i.e., a column of rock with thickness d and radioactive generation A contributes to the surface heat flow by an amount of Ad.

In the same way, if  $Q_d$  is the mantle heat flow, it contributes  $Q_d.z/K$  to the temperature at depth z.

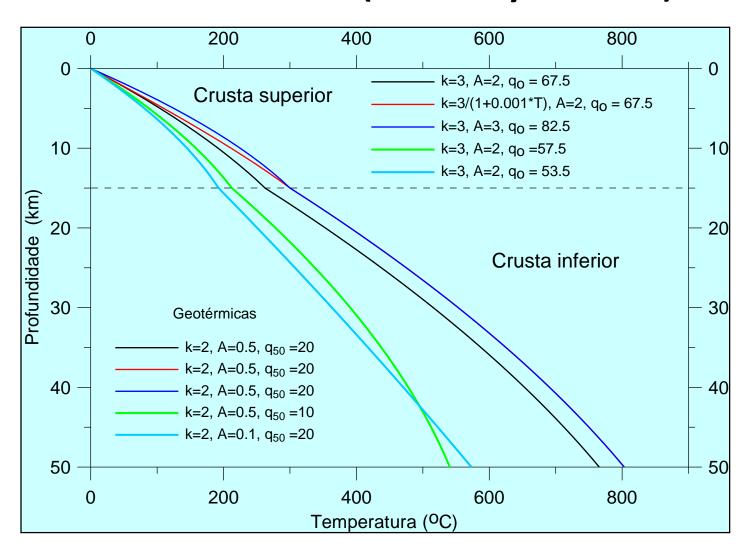
$$T = T_0 + \frac{Q_0}{K} \cdot z - \frac{A}{2K} \cdot z^2 \qquad T = \frac{Q_d + Ad}{K} \cdot z - \frac{A}{2K} \cdot z^2$$

## Geotherms (steady state)

$$T = T_0 - \frac{A_1}{2K} \cdot z^2 + \left(\frac{Q_2}{K} + \frac{A_2}{K}(z_2 - z_1) + \frac{A_1 \cdot z_1}{K}\right) \cdot Z \qquad (0 \le z < z_1)$$

$$T = T_0 - \frac{A_2}{2K} \cdot z^2 + \left(\frac{Q_2}{K} + \frac{A_2 \cdot z_2}{K}\right) \cdot z + \frac{A_1 - A_2}{2K} \cdot z_1^2 \qquad (z_1 \le z < z_2)$$

## Geotherms (steady state)



### Heat conduction equation

Of course we can consider other complications such as non stationary temperature (variation of temperature with time) and other boundary conditions

$$\frac{\partial T}{\partial t} = \alpha \cdot \frac{\partial^2 T}{\partial z^2} \qquad \text{with} \qquad \frac{\partial T}{\partial t} \neq 0$$

## Periodic temperature change at the Earth's surface

The heat conduction equation can be integrated to give the temperature distribution as a function of time (t) and depth (z)

$$\frac{\partial \mathbf{T}}{\partial \mathbf{t}} = \alpha \cdot \frac{\partial^2 T}{\partial z^2}$$

$$T(0,t) = T_0 \cdot e^{i\omega t}$$

$$T(z,t) \to 0 \quad \text{for} \quad z \to \infty$$

$$T(z,t) = T_{0} \cdot exp \left( -\sqrt{\frac{\omega \cdot \rho \cdot c_{p}}{2K}} \cdot z \right) \cdot exp \left[ i \left( \omega \cdot t - \sqrt{\frac{\omega \cdot \rho \cdot c_{p}}{2K}} \cdot z \right) \right]$$

$$L = \sqrt{\frac{2K}{\omega \cdot \rho \cdot c_{P}}} \qquad \Phi = \sqrt{\frac{\omega \cdot \rho \cdot c_{P}}{2K}}$$

## Periodic temperature change at the Earth's surface

- For a sinusoidal temperature perturbation at the surface, the temperature variation for large depths tends to zero.
- For a depth of L= $(2K/\omega\rho c_p)^{1/2}$ , the temperature perturbation has an amplitude of 1/e of the amplitude at the surface. L is called the skin depth.
- For a daily temperature variation L=17 cm, for an annual temperature variation L=3.3 m, and for time periods of the order of 10<sup>5</sup> years L>1 km.

#### Problem diversion

- 1. Calculate the angular frequency for a sinusoidal temperature perturbation of one year.
- 2. Show that for a depth of  $z=(2K/\omega\rho c_p)^{1/2}$ , the temperature perturbation has an amplitude of 1/e of the amplitude at the surface.
- 3. Show that for an annual temperature variation the skin depth is 3.3 m.

#### Problem diversion

- 1. Calculate the angular frequency for a sinusoidal temperature perturbation of one year.
- 2. Show that for a depth of  $z=(2K/\omega\rho c_p)^{1/2}$ , the temperature perturbation has an amplitude of 1/e of the amplitude at the surface.
- 3. Show that for an annual temperature variation the skin depth is 3.3 m.

Take K=2.5 W/mK,  $c_p=103$  J/kg.°C and  $\rho=2.3$  x  $10^3$  kg/m<sup>3</sup> (approximate values for sandstone).

$$\omega = 2 \cdot \pi \cdot f = 1.9 \times 10^{-7} \text{ s}^{-1}$$

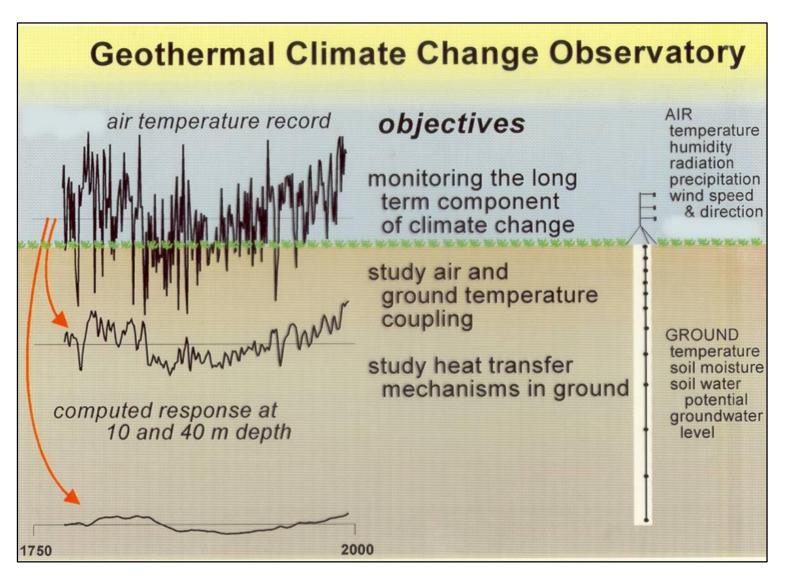


Geothermal Climate
Change Observatory
in the TGQC-1 well
(borehole 190 m deep)

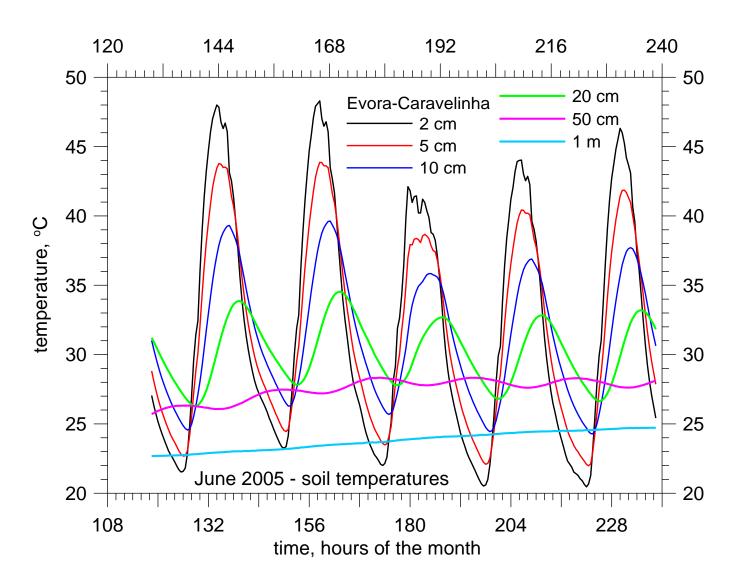
## Depth (m) of sensors in the borehole:

0.02 0.05 0.10 0.20 0.50 1.0 2.0 5.0 10.0 20.0 30.0 e 40.0

### Geothermal paleoclimatology

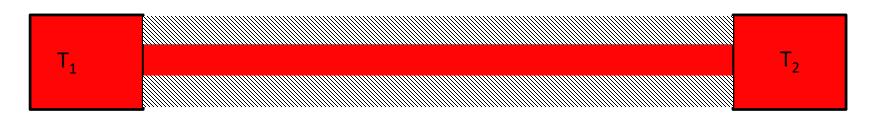


## Periodic temperature change at the Earth's surface



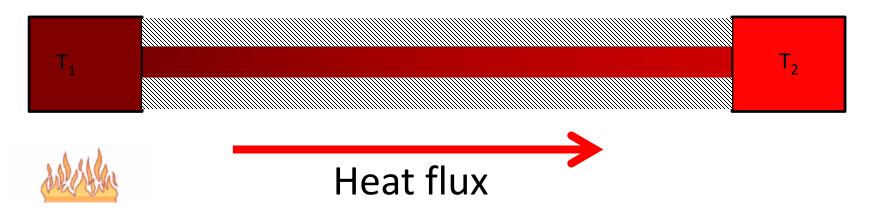
$$T_1 = T_2$$

#### Insulation



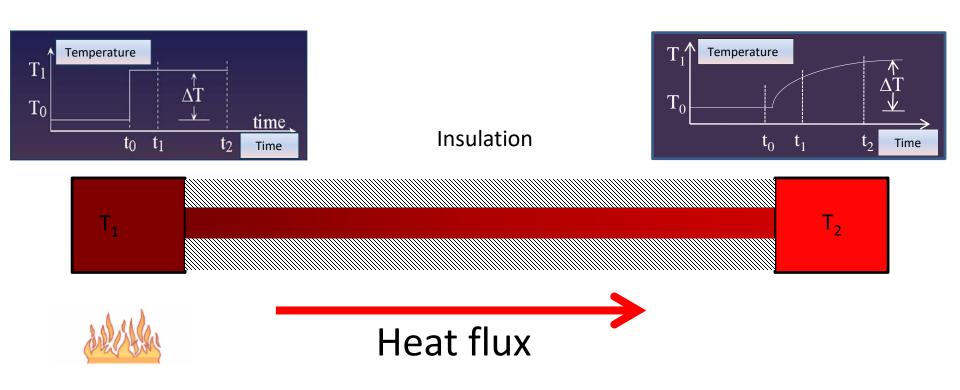
$$T_1 > T_2$$

#### Insulation

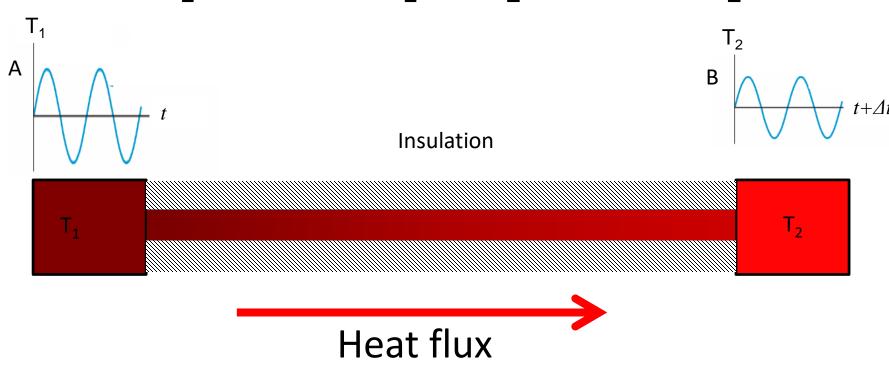


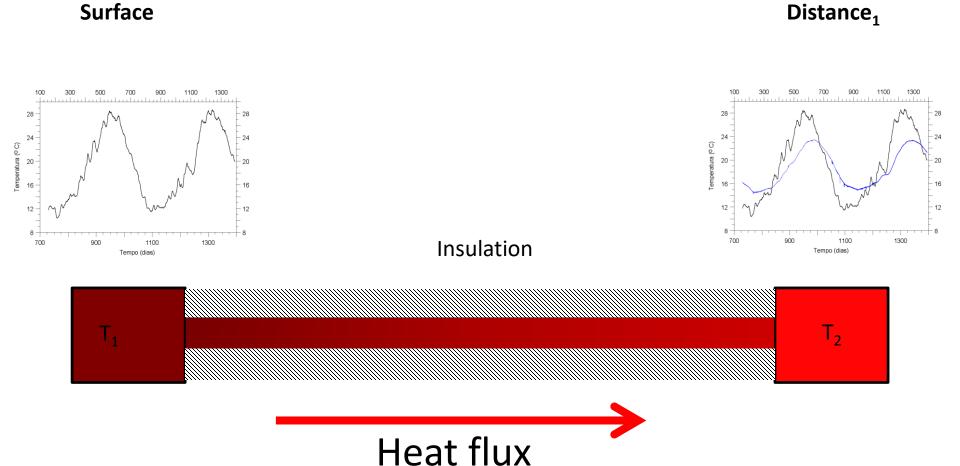
Temperature jump of  $\Delta T$ 

For very long times temperature tends to  $T_1$ 

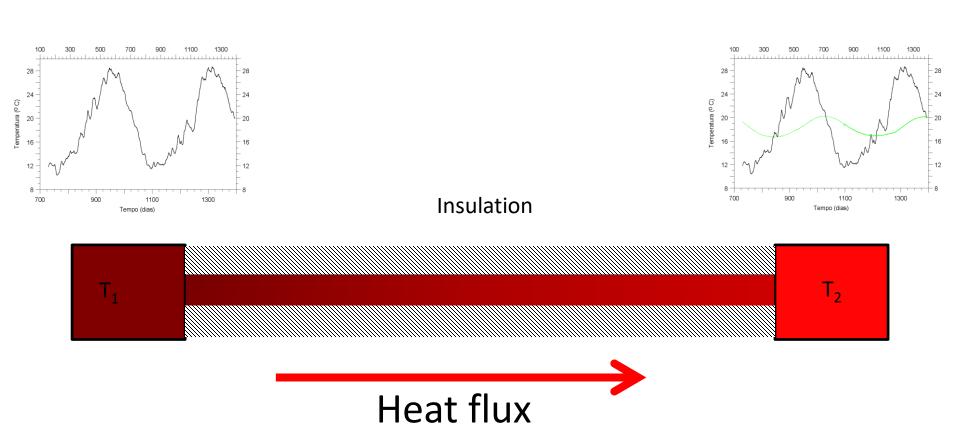


$$T_1 = A \cdot sen(\omega t_1) > T_2 = B \cdot sen(\omega t_2)$$

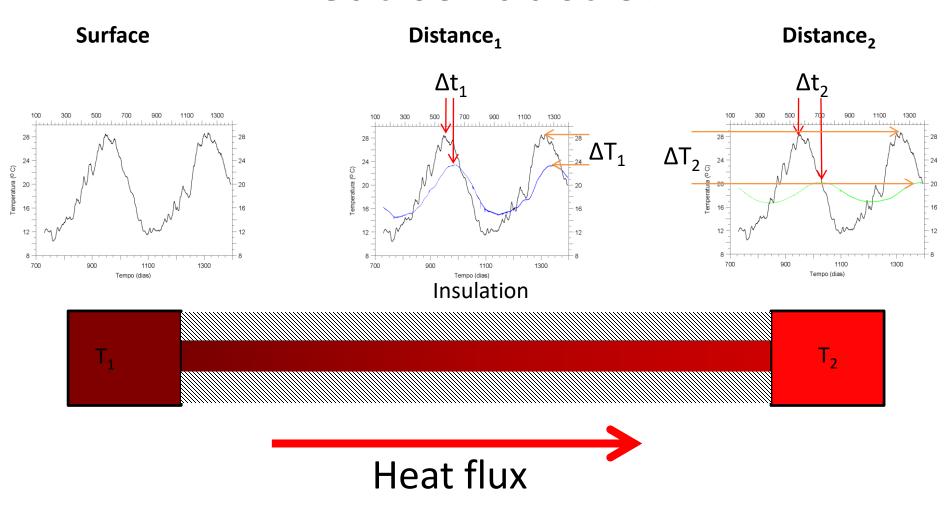




Surface Distance<sub>2</sub>

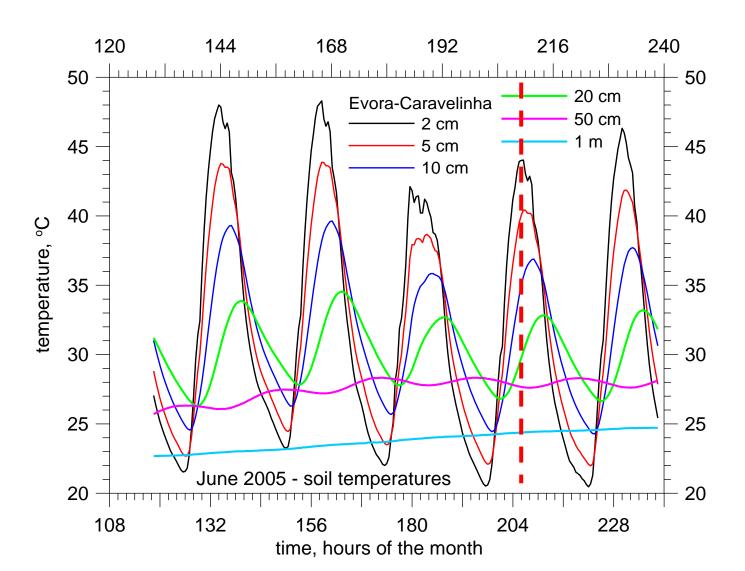


Distance<sub>1</sub> < Distance<sub>2</sub>



 $\Delta t_2 > \Delta t_1$  e  $\Delta T_2 > \Delta T_1$ 

## Periodic temperature change at the Earth's surface



#### Some obvious conclusions

- High frequency components are attenuated
- Signal decreases in amplitude with distance (depth)
- The time lag increases with distance (depth)
- The signal's wavelength increases with distance (depth)
- As distance (depth) increases resolution decreases

## Temperature change: cooling phenomena in the Earth

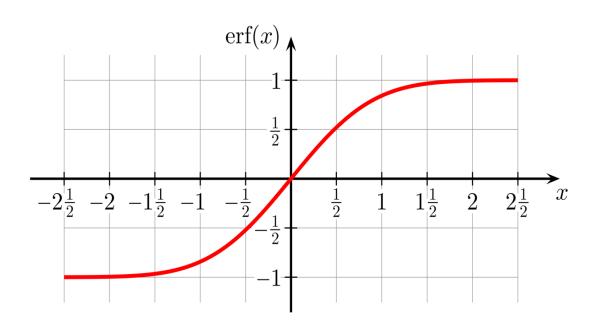
$$\frac{\partial T}{\partial t} = \alpha \cdot \frac{\partial^2 T}{\partial z^2} \qquad \alpha = \frac{K}{\rho \cdot c_p}$$

Semi-infinite solid with surface at z=0 and initial temperature  $T=T_0$ Solution for t>o with surface temperature T=0

$$T = T_0 \cdot erf\left(\frac{z}{2\sqrt{\alpha \cdot t}}\right) \qquad erf\left(x\right) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy$$

# Temperature change: cooling phenomena in the Earth

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-y^{2}} dy$$



# Temperature change: cooling phenomena in the Earth

Differentiating in oder to z

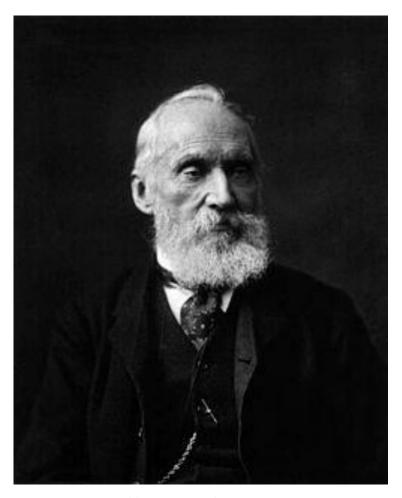
$$T = T_0 \cdot erf\left(\frac{z}{2\sqrt{\alpha \cdot t}}\right)$$

$$\frac{\partial \mathbf{T}}{\partial \mathbf{z}} = \frac{\partial}{\partial \mathbf{z}} \left( \mathbf{T}_0 \cdot \operatorname{erf} \left( \frac{\mathbf{z}}{2\sqrt{\alpha \cdot \mathbf{t}}} \right) \right) = \frac{\mathbf{T}_0}{\sqrt{\pi \cdot \alpha \cdot \mathbf{t}}} \cdot e^{-\mathbf{z}^2/4\alpha \mathbf{t}}$$

which for z=0 gives

$$\left(\frac{\partial \mathbf{T}}{\partial \mathbf{z}}\right)_{z=0} = \frac{\mathbf{T}_0}{\sqrt{\pi \cdot \alpha \cdot \mathbf{t}}}$$

### Lord Kelvin and the age of the Earth



William Thomson (26 June 1824 – 17 December 1907)

$$t = \frac{T_0^2}{\left(\frac{dT}{dz}\right)_{z=0}^2 \cdot \pi \cdot \alpha}$$

$$T_0 = 3871 \, ^{\circ}\text{C} \quad (= 7000 \, ^{\circ}\text{F})$$

$$(dT/dz)_{z=0} = 0,036 \, ^{\circ}\text{C m}^{-1}$$

$$\alpha = 1,4 \times 10^{-6} \, \text{m}^2 \, \text{s}^{-1}$$

$$t = 100 \, \text{Ma}$$

### Lord Kelvin and the age of the Earth

Kelvin argued that a positive thermal gradient with depth means that the Earth must be cooling. By assuming the Earth began as a sphere initially at a constant temperature, he was able to calculate when the cooling must have started. Using an average gradient of 36.5°Ckm<sup>-1</sup> (l°F/50 ft), he calculated the age of the Earth to be 200 Ma if the initial temperature was 5540°C (10,000°F), or only 98 Ma if the initial temperature was 3870°C (7000°F).

Kelvin preferred the latter estimate, but included a sizeable error margin, 20-400 Ma, due to the uncertainty in the value of thermal diffusivity for crustal rocks (Thomson, 1862).

## Thermal history of the Earth

The firt attempt to estimate the Earth's age was done by William Thomson (Lord Kelvin)

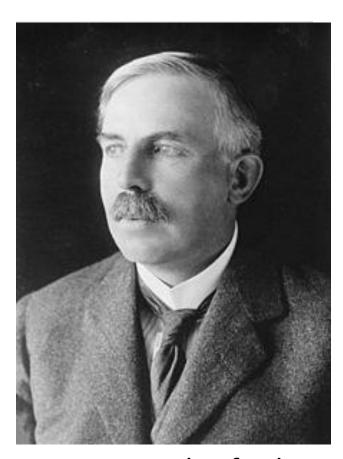
$$t = \frac{T_0^2}{\left(\frac{dT}{dz}\right)_{z=0}^2 \cdot \pi \cdot \alpha}$$

$$T_0 = 3871 \, ^{\circ}\text{C} \quad (= 7000 \, ^{\circ}\text{F})$$

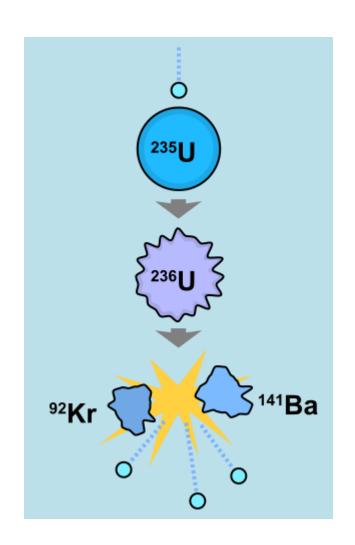
$$(dT/dz)_{z=0} = 0,036 \, ^{\circ}\text{C m}^{-1}$$

$$\alpha = 1,4 \times 10^{-6} \, \text{m}^2 \, \text{s}^{-1}$$

$$t = 100 \, \text{Ma}$$



Ernest Rutherford (30 August 1871 – 19 October 1937)

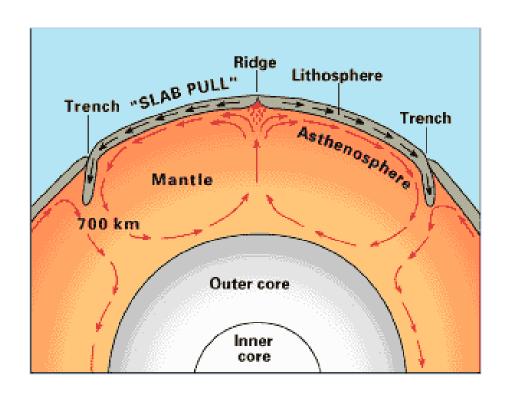


- Major modifications had to be made to Kelvin's model after it was recognized that radiogenic heating acted to significantly extend the cooling time.
- Rutherford began the job; serious age estimates were developed after models of the distribution of radioactive isotopes within the Earth suggested by Lubimova (1958) and MacDonald (1959).

- Turcotte (1980) attributed 83% of the present surface heat flow to the decay of radioactive isotopes, and only 17% to the cooling of the Earth. He concluded that the mantle is presently cooling at a rate of 36 °C Ga<sup>-1</sup>, and that three billion years ago it was likely 150 °C hotter than at present.
- The presently accepted age of the Earth is around 4.55 Ga.



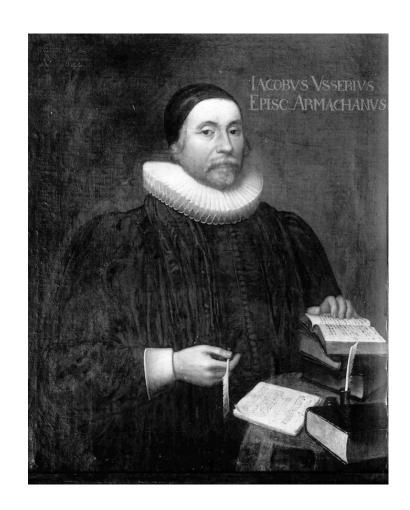
John Perry (14 February 1850 – 4 August 1920)



John Perry, a former Kelvin assistant realized that some kind of convection should be taking place inside the Earth. However, he hesitated to criticize Kelvin. Finally, long after Kelvin made his prediction, Perry went to him with a convection-based idea. Kelvin brushed him off. So, in 1895, Perry published his idea in Nature.

Kelvin' age for the Earth was wrong because radioactivity in Earth's mantle released energy. That makes the gradient steeper and Earth seems younger. However, Kelvin did not use the wrong gradient! He used a completely wrong physical model. We now know that both convection and conduction moves heat through Earth's interior.

In 1650, the Irish
Archbishop James Ussher
(1581-1656) announced
that according to the Old
Testament, the Earth began
during the "beginning of
the night of October 22nd
of the year 4004 b.C."



## II Geothermal gradient

### Reminder

$$q_z = -\frac{\Delta Q}{A \cdot \Delta t} = -K \frac{dT}{dz}$$

 $q_z$  is a vector as well as the temperature gradient dT/dz. To calculate (estimate) the heat flux we have to know thermal conductivity K and geothermal gradient dT/dz. Most of the times a  $q_z$  vertical is assumed.

### Temperature logs

A temperature log is a direct measure of the actual temperature in a borehole. It can be obtained in land or offshore.

- In land, water, mining or oil wells can be used for temperature measurement (with different degrees of accuracy)
- Offshore, temperatures are measured with special probes (violin-bow type probes) inserted by gravity in soft sediments or using boreholes of the late ODP (Ocean Drillig Program) or IODP (Integrated Ocean Drillig Program)

### Temperature logs

Temperature logs are used to estimate the geothermal gradient, which, by definition is given by:

$$\nabla T = \text{grad } T = \frac{dT}{dz} = \frac{T_2 - T_1}{\Delta z}$$

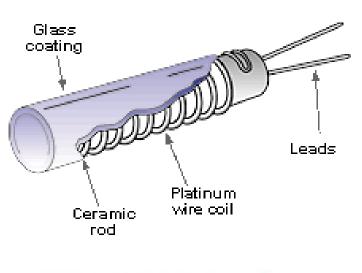
where  $T_1$  (shallower) and  $T_2$  (deeper) are the temperatures at two points separated by a distance  $\Delta z$ . It is vector quantity and has magnitude and direction; by convention, it's positive in the direction of increasing temperature. In SI is expressed in K/m or Km<sup>-1</sup> (kelvin per meter). Practical unit is  ${}^{\circ}$ C/km.

### Direct temperature measurements

#### **Precision Temperature Logs**

- Thermistors
- Thermo-couples (out dated)
- Platinum resistances
- DTS (Distributed optical fibre Temperature Sensing system)

They can be programmable computer loggers or probes connected to the surface by conductors

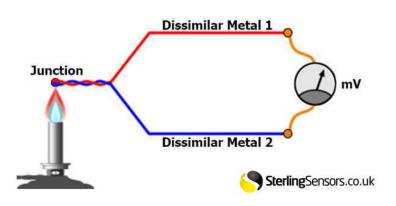


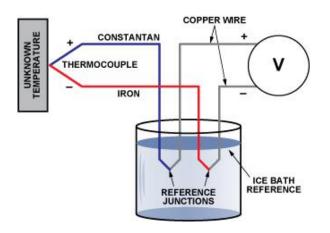


Platinum resistance

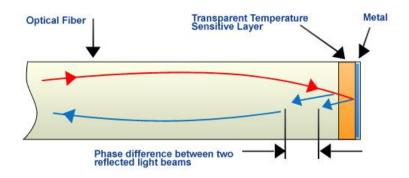


**Thermistors** 





Fiber Optic Temperature Sensor Using Phase Interference



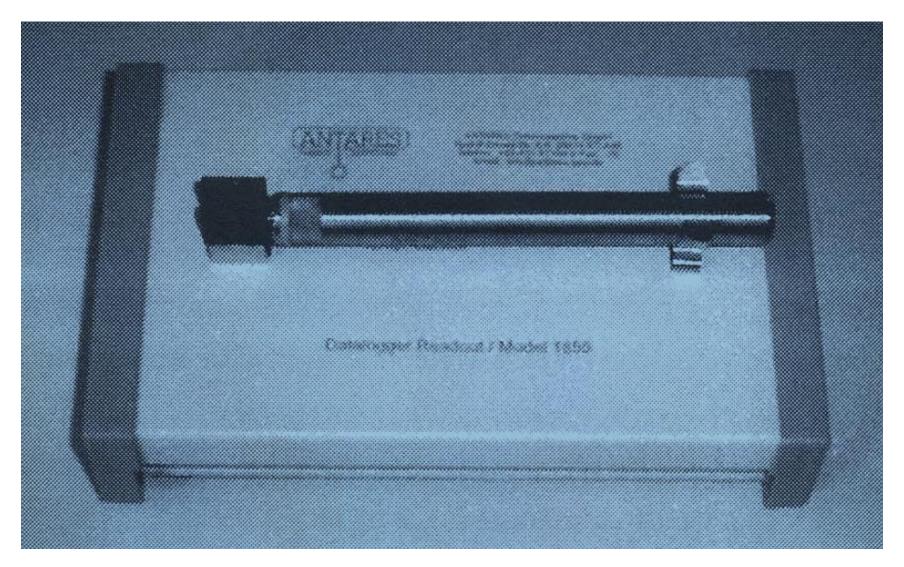
© 2010 Chipkin Automation Systems Inc.

**DTS** 

Thermocouples



ANTARES Datalogger. SN-185 42 50 and protection casing







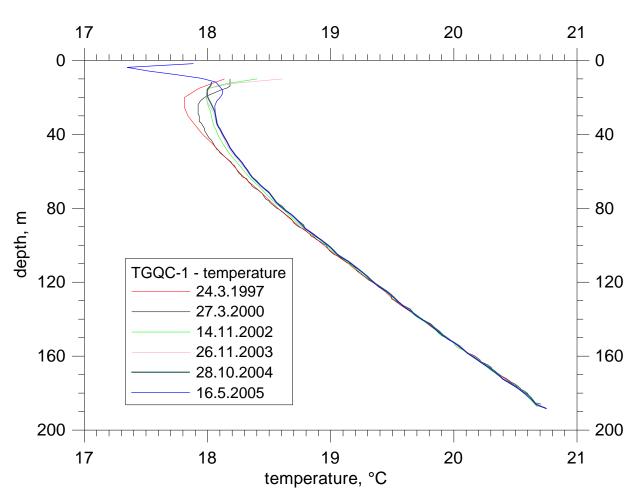
Geothermal Climate Change Observatory in the TGQC-1 well



Temperature logs for different years



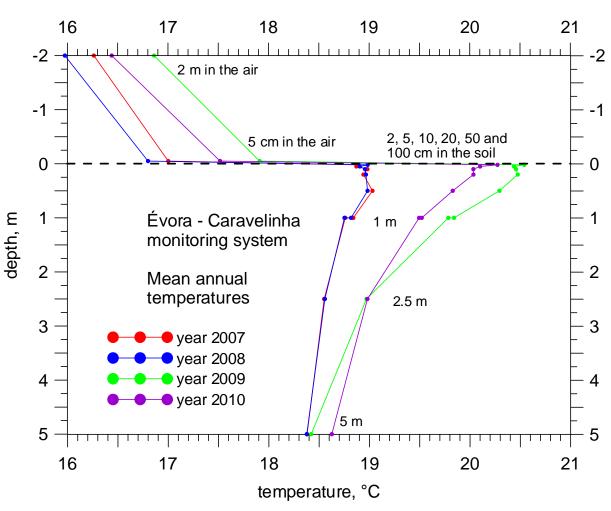
Geothermal Climate Change Observatory in the TGQC-1 well



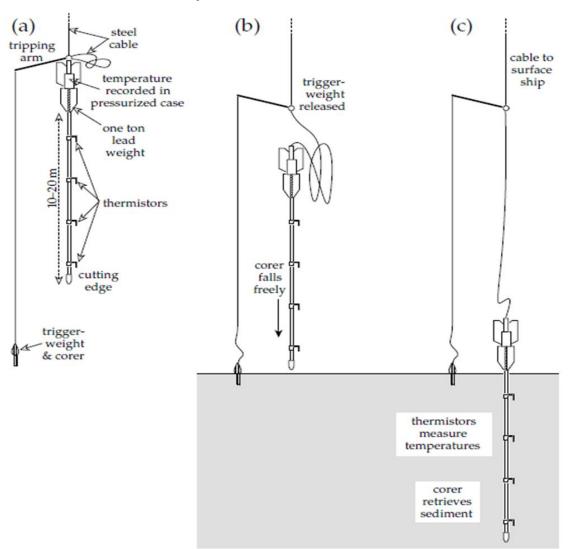
Temperatures at the surface of the ground



Geothermal Climate Change Observatory in the TGQC-1 well



#### **Deep-Water Probes**



# Problems with temperature measurements

When mesuring temperatures in boreholes a few problems have to be considered and corrections may have to be applied:

- Borehole convection
- BHT correction

# Problems with temperature measurements: borehole convection

A column of fluid under the influence of a thermal gradient may support convection cells which may cause the temperature log within the fluid to differ from that within the surrounding rocks.

# Problems with temperature measurements: borehole convection

Critical thermal gradient above which convection start:

$$\frac{\partial T}{\partial z} = \frac{C \cdot \upsilon \cdot \kappa}{g \cdot \alpha \cdot r^4} \qquad (°C/m \text{ or } K/m)$$

where  $\frac{\partial T}{\partial z}$  is the critical thermal gradient; g is the acceleration of gravity;  $\alpha$ , v and  $\kappa$  are the thermal expansion coefficient, the kinematic viscosity and the thermal diffusivity of the fluid; r is the borehole radius; C is a constant equal to 2.16 x  $10^{-4}$  (if SI units are used).

# Problems with temperature measurements: borehole convection

For water at 95 °C

$$\frac{\partial T}{\partial z} = \frac{C \cdot \upsilon \cdot \kappa}{g \cdot \alpha \cdot r^4}$$

reduces to approximately

$$\frac{\partial T}{\partial z} = \frac{1.4 \cdot 10^{-9}}{r^4}$$

What are the units of the constant 1.4 x 10<sup>-9</sup> in the second equation?

#### Diversion about units

$$[g] = m \cdot s^{-2} \qquad [\alpha] = K^{-1} \qquad [\nu] = m^2 \cdot s^{-1}$$
$$[\kappa] = m^2 \cdot s^{-1} \qquad [r] = m$$
$$\frac{\partial T}{\partial z} = \frac{C \cdot \upsilon \cdot \kappa}{g \cdot \alpha \cdot r^4}$$

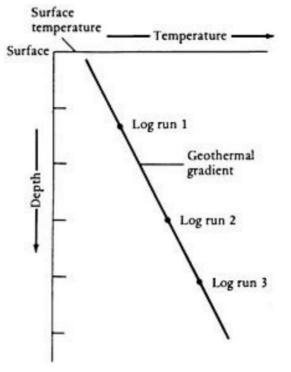
What are the units of constant  $C = 2.16 \times 10^{-4}$ ?

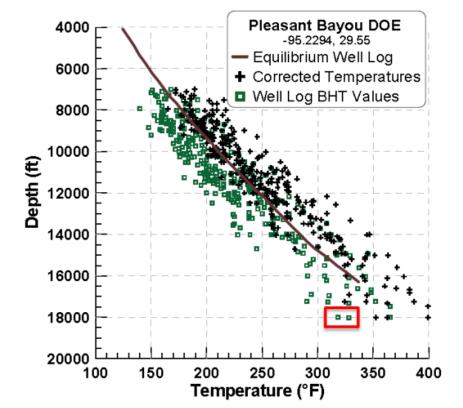
Verify the dimensional homogeneity of the equation

**Bottom-Hole Temperatures (BHT)** 

They are obtained by attaching maximum thermometers

to probes in different runs in oil prospecting wells





#### The Horner plot method for correcting BHTs

- 1. For <u>each depth</u> there must exist, at least, 2 BHTs measured at 2 different times.
- 2. It is necessary to know the time of the mud circulation in the borehole after stopping drilling  $(t_c)$ .
- 3. It is necessary to know the time elapsed between the end of the mud circulation and the measurement of temperature at the bottom of the borehole ( $\Delta t$ ).

#### The Horner plot method for correcting BHTs (cont.)

- 4. Therefore, for each BHT there must be a different time of measurement ( $\Delta t$ ).
- 5. A graph of BHT vs. In  $(1+t_c/(\Delta t))$  should be drawn.
- 6. The best fit straight line should be adjusted to the points in the graph.
- 7. Extrapolation of the straight line to the vertical axis (temperature axis) will give the geological formation temperature before drilling (VRT Virgin Rock Temperature).

#### **Example:**

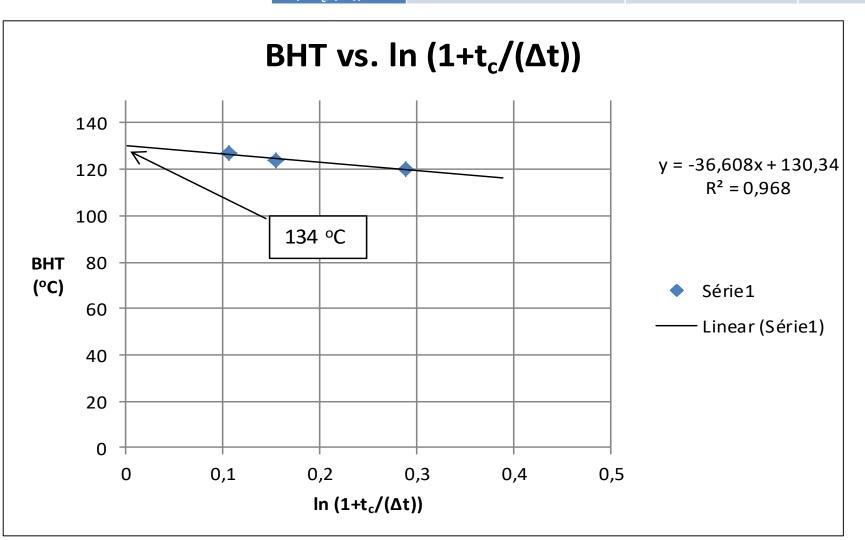
Borehole: Cormorant 1

Depth of the first runs of logs: 900 m.

Number of log runs: 3

Cormorant 1				
Depth – 900 m	SP/Res/GR run	Neutron run	64' N run	
Circulation Time (hours)	2	2	2	
BHT (°C)	120	124	127	
Time of meas. (Δt) (hours)	6	12	18	
In (1+t <sub>c</sub> /(Δt))	0.28768	0.15415	0.10536	

Cormorant 1				
Depth – 900 m	SP/Res/GR run	Neutron run	64' N run	
Circulation Time (hours)	2	2	2	
BHT (°C)	120	124	127	
Time of meas. (Δt) (hours)	6	12	18	
In (1+t <sub>c</sub> /(Δt))	0.28768	0.15415	0.10536	

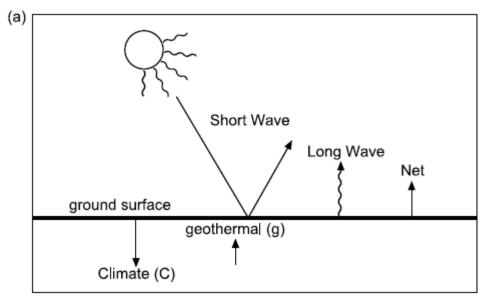


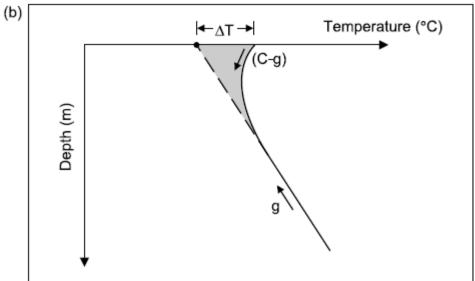
Bottom-Hole Temperatures

To make experiments with bottom hole temperatures you can go to this site:

http://zetaware.com/utilities/bht/horner.html

## A climate signal in ground temperature

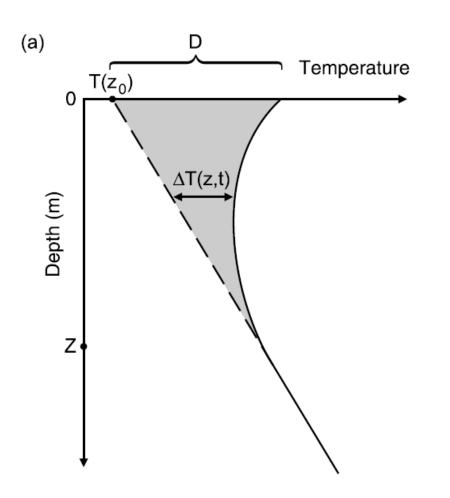


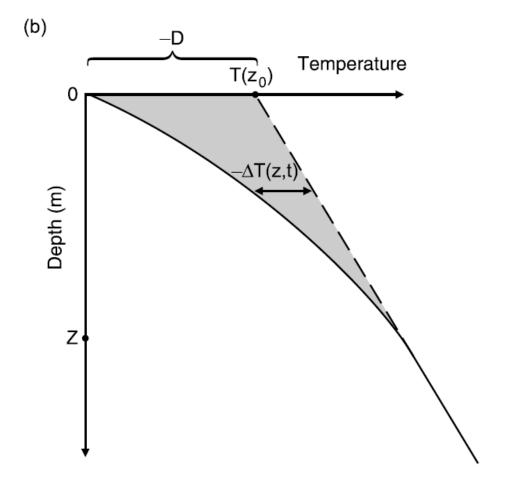


### The shape says a lot about climate

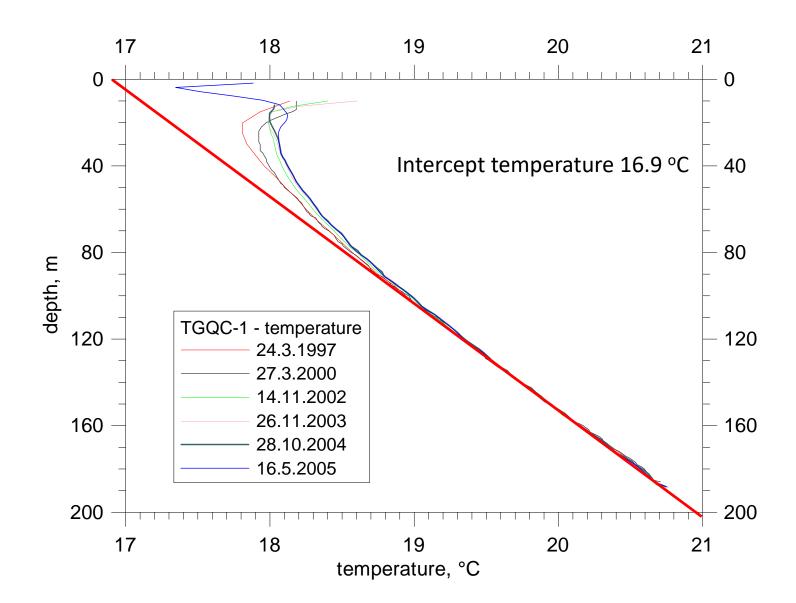
Increase of air temperature

Decrease of air temperature





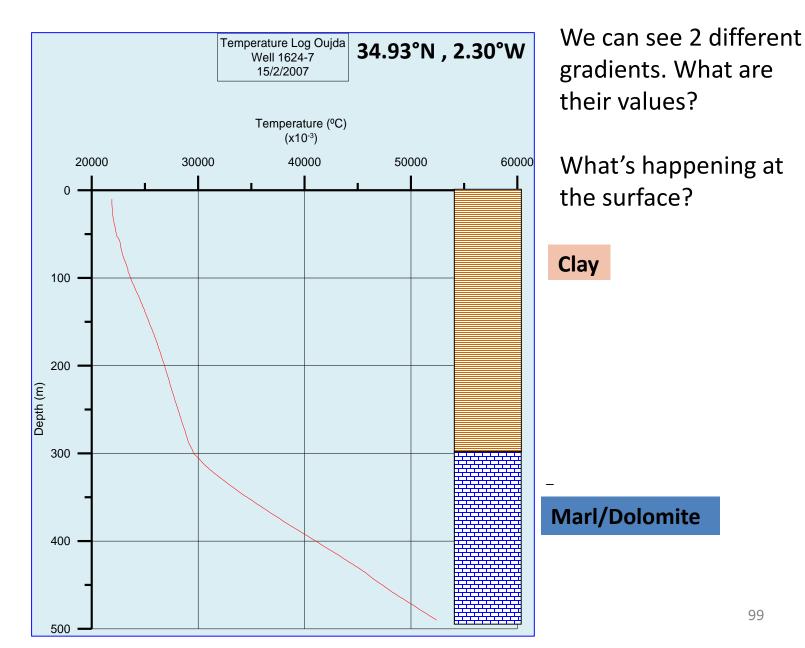
### Temperature logs for different years

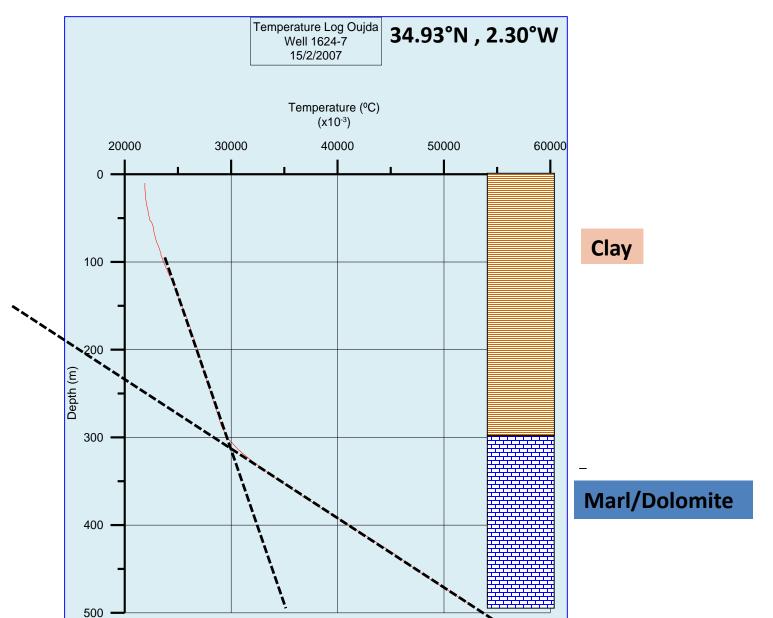


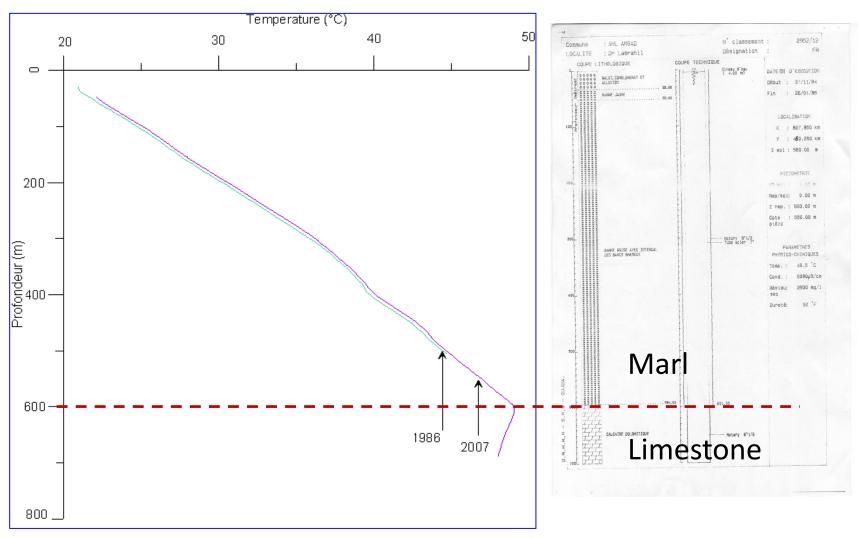


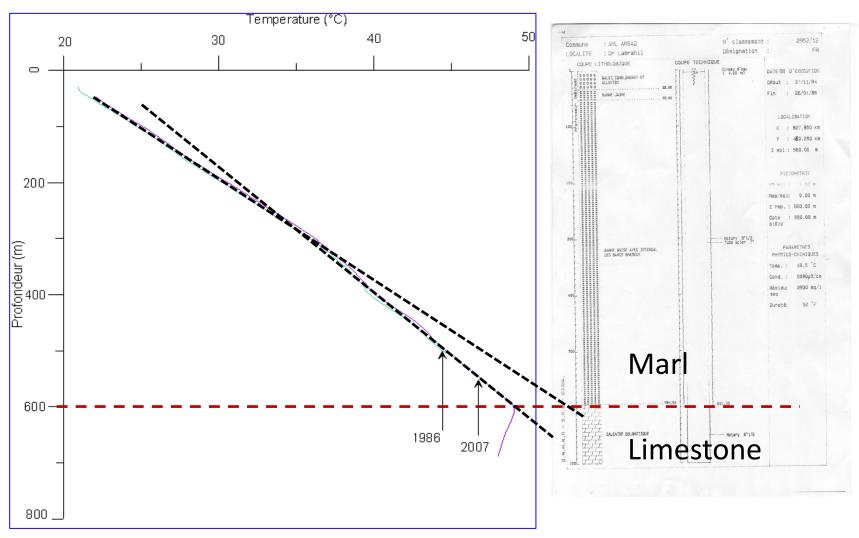
For the next examples try:

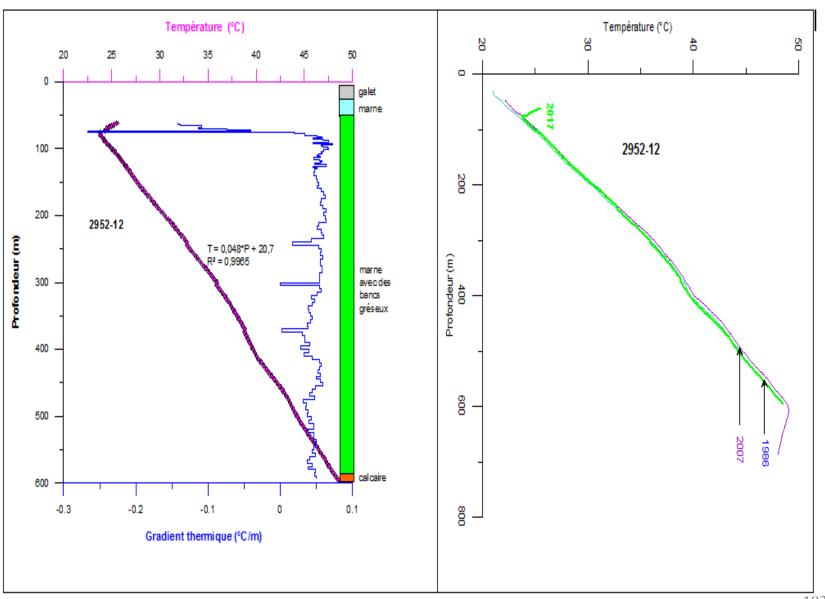
- To estimate the geothermal gradient
- Give plausible interpretion of the geological, geophysical and hydrodynamical situation







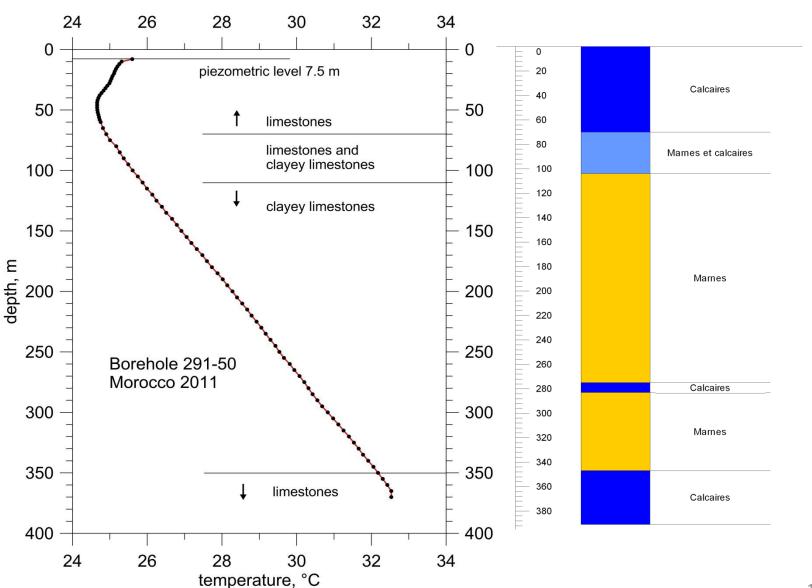


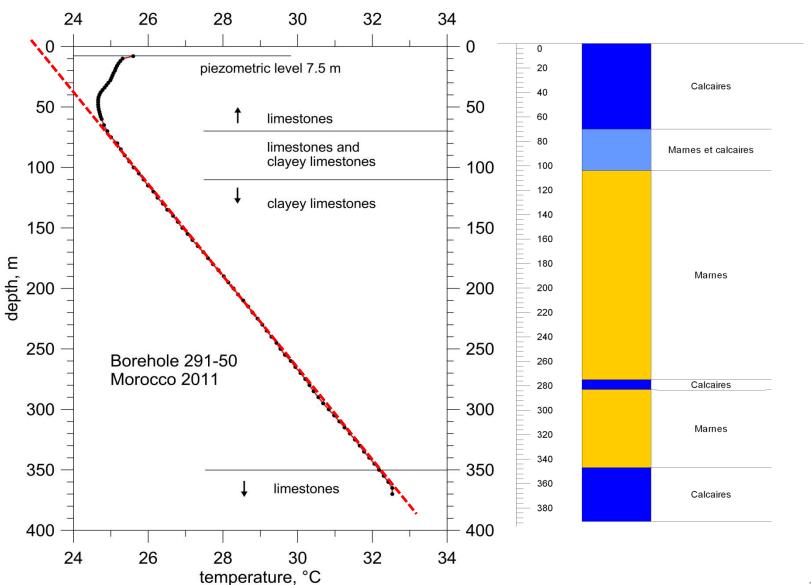




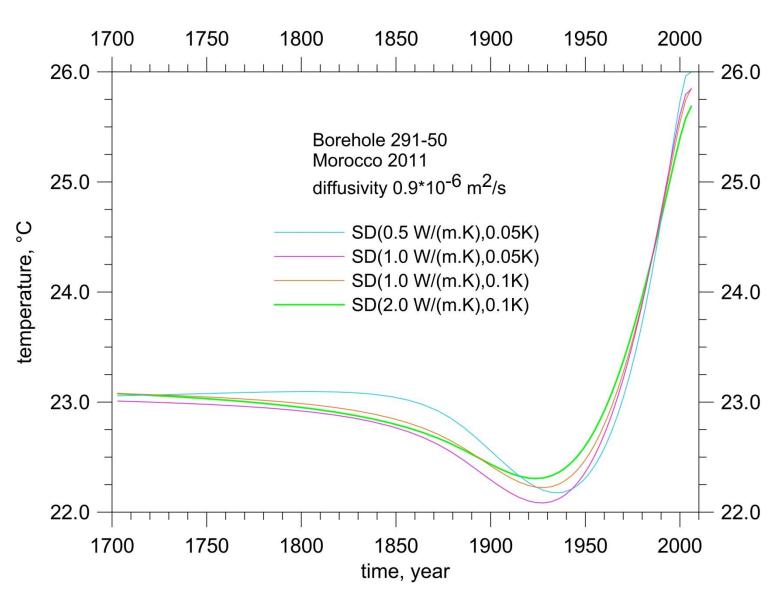








# Examples (Morocco)

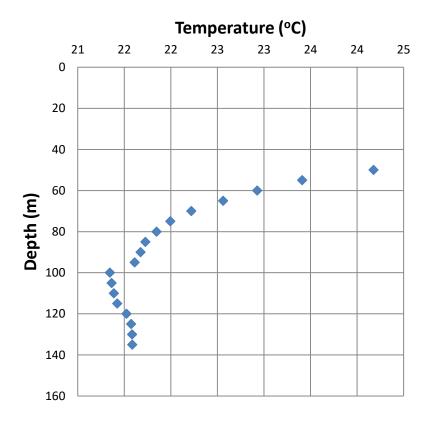


# Examples (Morocco)

#### **Borehole 2802/12**

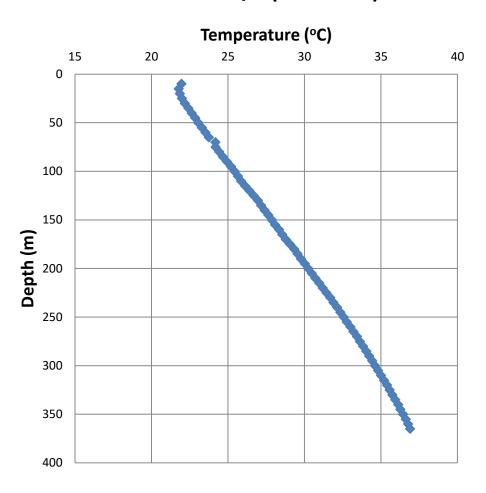
#### Temperature (°C) Depth (m)

#### **Borehole 3394/12**

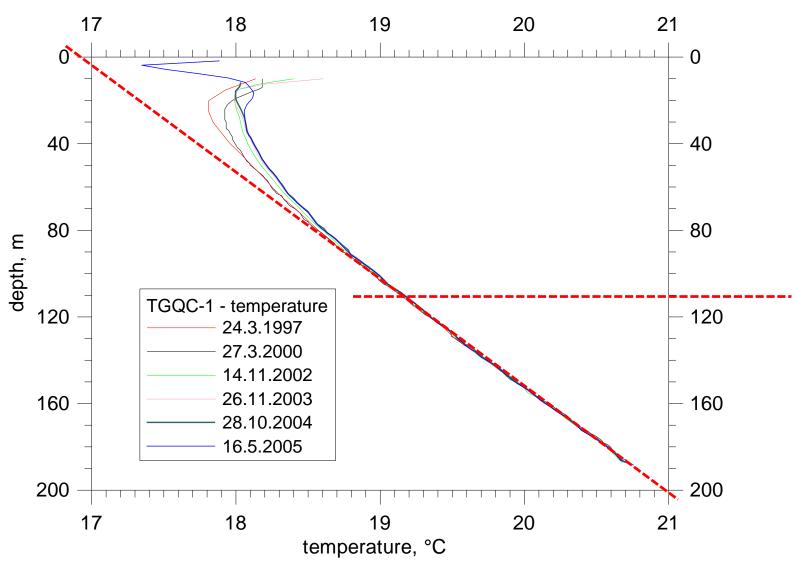


# Examples (Morocco)

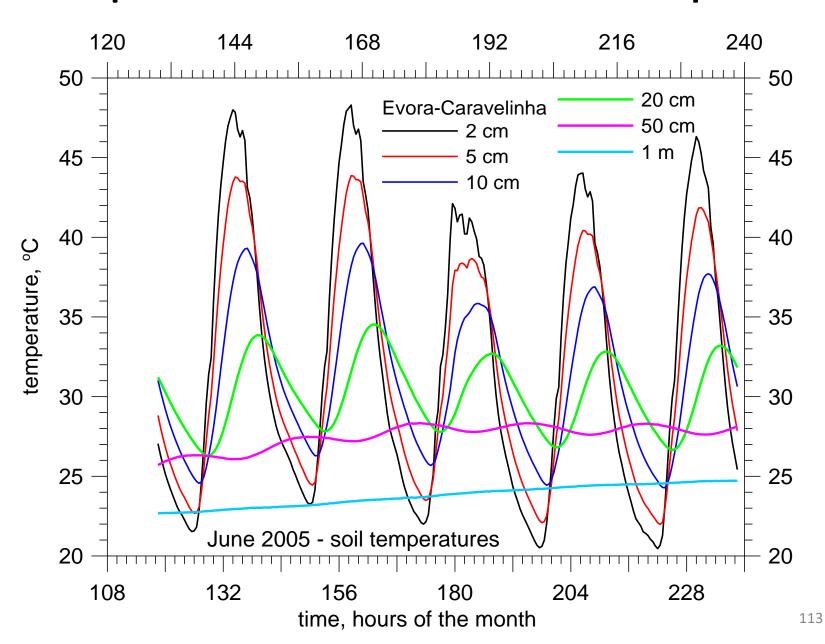
#### **Borehole 1631/7 (Fezouane)**



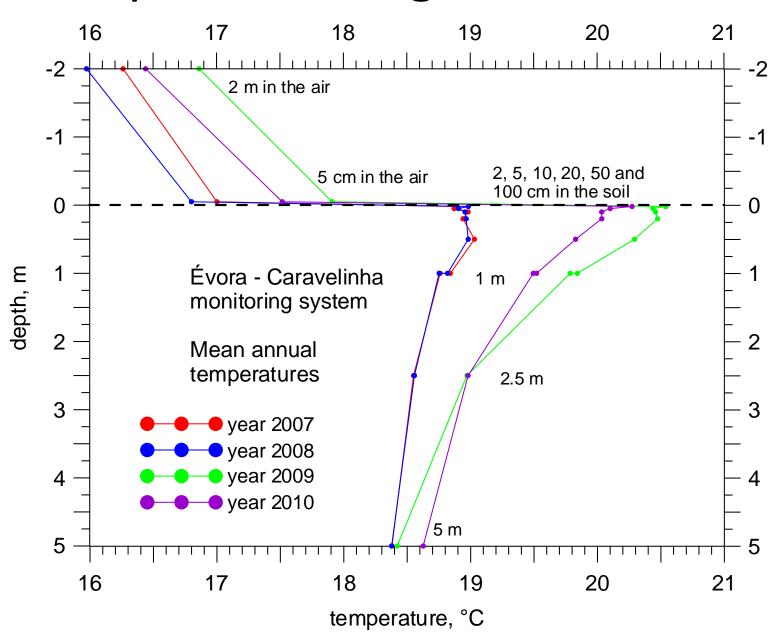
# Temperature logs for different years



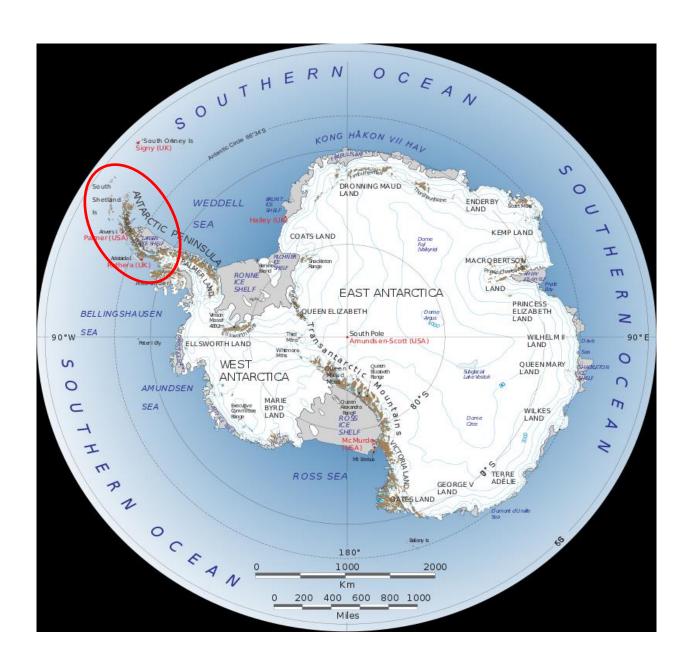
### Temperature for different depths



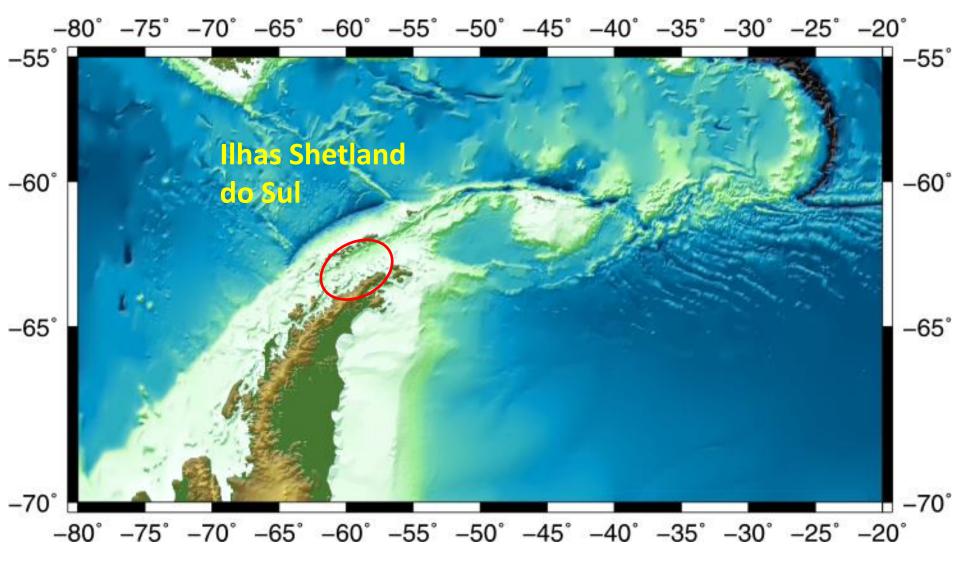
# Temperature at ground's surface



#### The Antarctica Continent

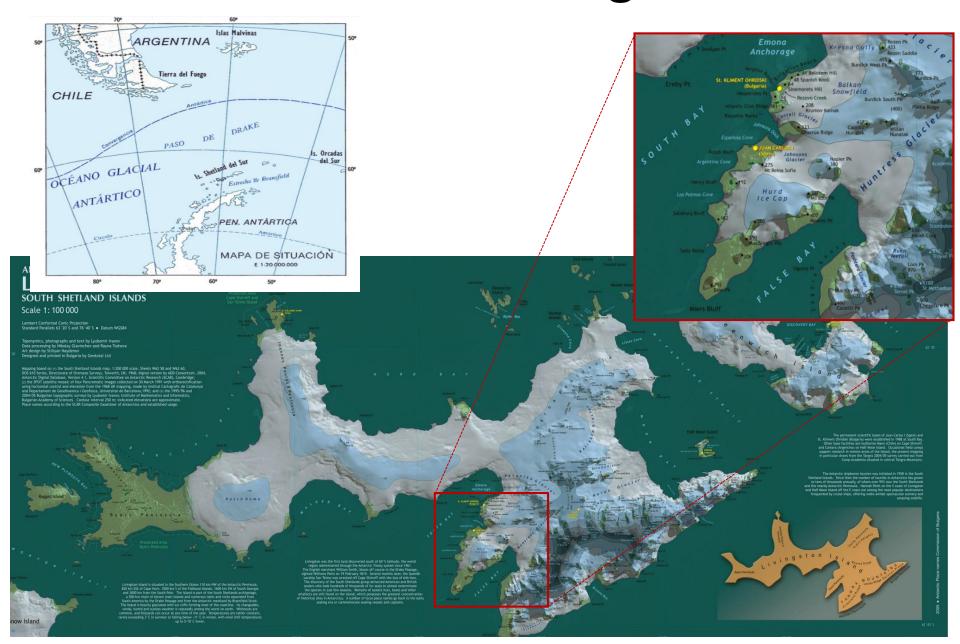


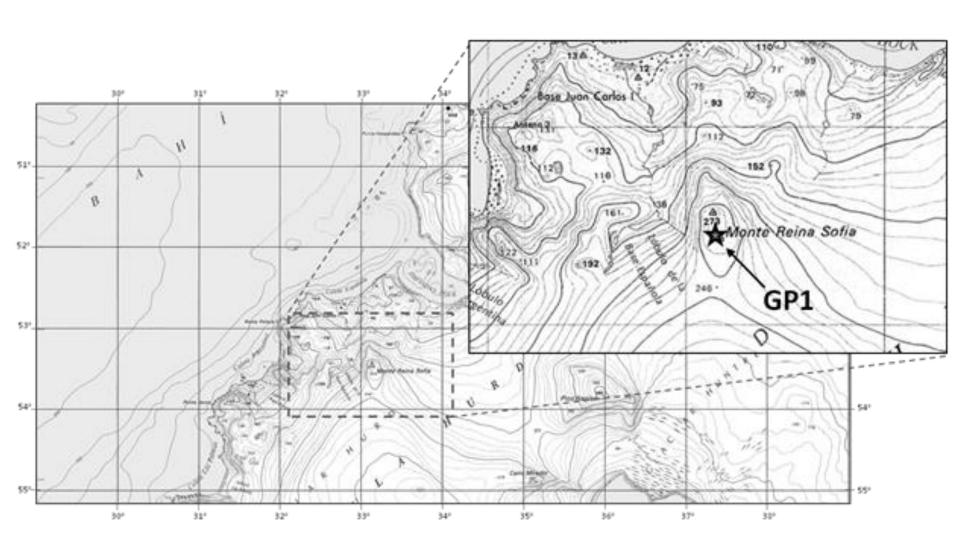
#### Antarctica Peninsula



# Livingston and Deception Islands

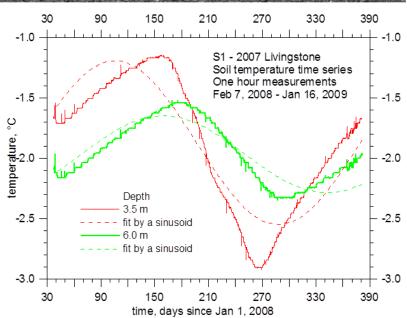


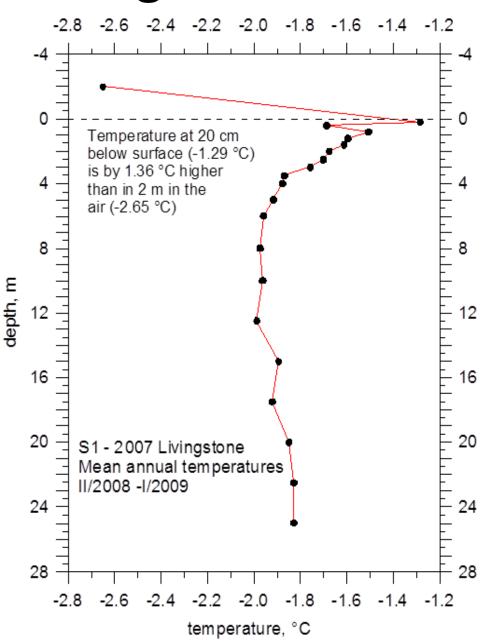


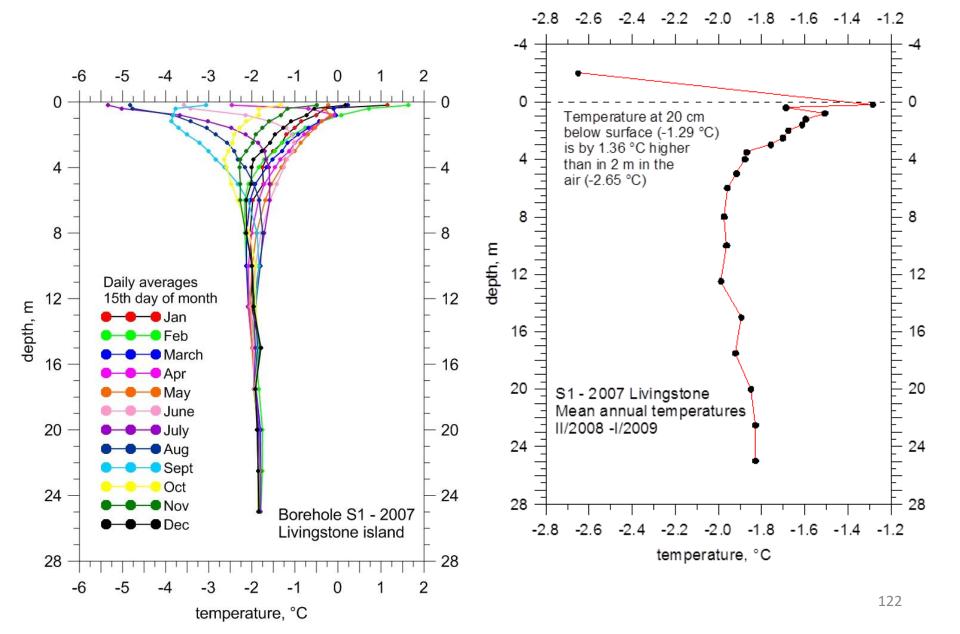




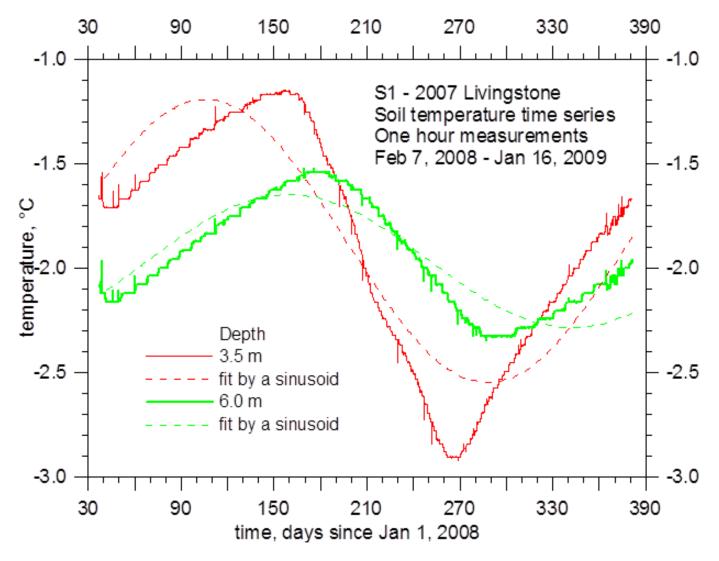








Why the real data does not fit the sinusoids?



### Indirect temperature measurements

As a complement to direct methods to measure temperature, other indirect methods have been developed. However, in all of them the accuracy is much lower than in the direct methods. For heat flow density studies in the crust they serve as indicators of its thermal regime. They are:

- Groundwater Geochemistry
- Curie Depth
- Xenoliths
- Upper Mantle Resistivity

The solubility of many compounds in water increases with temperature. The temperature of a given porous formation can be estimated from the amount of dissolved material in its water. Relationships of this type (geothermometers) have been developed for several components or chemical elements present in the water filling the pores of different rocks. For silica ( $SiO_2$ ) an empirical expression was developed by Swanberg and Morgan (1979):

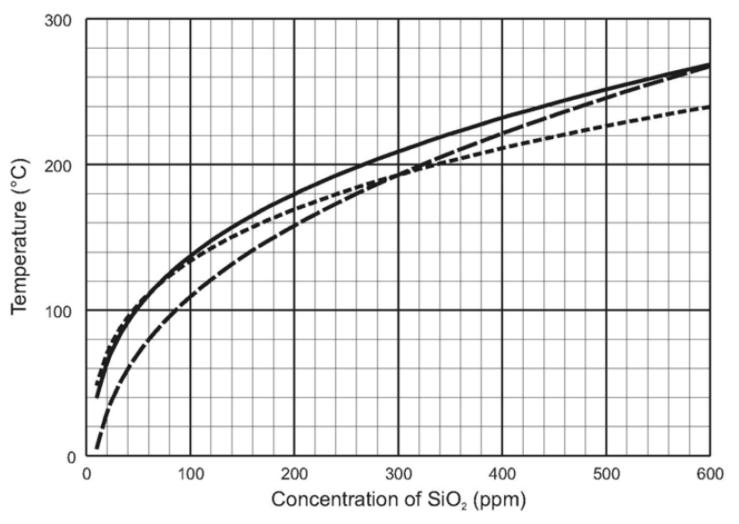
$$T = [1315/(5.205 - \log_{10}(SiO_2))] \pm 0.5^{\circ}C \quad (1)$$

where T is the estimated water temperature (K) and SiO<sub>2</sub> is the amount of dissolved silica (ppm) in the formation water. The quoted precision is valid in the range 125-250 °C.

Other thermometers using silica where developed such as:

$$T = [1533.5/(5.768 - log_{10}(SiO_2))] \pm 2.0 ^{\circ}C$$
 (2)

$$T = [1015.1/(4.655 - \log_{10} (SiO_2))] \pm 2.0 \,^{\circ}C \quad (3)$$



Relationships between concentration of dissolved silica in groundwater (ppm) and temperature of the water (°C). Solid line - Eq. (1); short-dashed line - Eq. (2); long-dashed line - Eq. (3).

Geothermometers for sodium (Na), potassium (K) and calcium (Ca),

$$T = [855.6/(0.8573 + log10(Na/K))] \pm 2.0 ^{\circ}C$$
 (4)

where Na and K are concentrations in ppm and temperature is in kelvin. Fournier and Truesdell (1973) suggested the following:

$$T = [846/(0.5964 + \log 10(M_{Na}/M_{K}))]$$
 (5)

$$T = [1647.3/(2.24 + \log_{10}(M_{Na}/M_{K}) + \beta \cdot \log_{10}(\sqrt{M_{Ca}}/M_{Na}))]$$
 (6)

where  $M_x$  is the molar concentration of element X (moles per litre),  $\beta$  is 4/3 for  $(\sqrt{M_{Ca}}/M_{Na}) > 1$  and T < 373.15 K (100°C),  $\beta$  is 1/3 for  $(\sqrt{M_{Ca}}/M_{Na}) < 1$  or T > 373.15 K (100°C), and T is in kelvin

Geothermometers are most useful in hightemperature geothermal reservoirs where direct temperature measurements are not possible or are difficult. As a matter of fact, they were developed for use in highenthalpy geothermal reservoirs

### Curie Depth

By definition, the *Curie temperature* is the point at which a mineral loses its ferromagnetic properties and the *Curie depth* is the depth at which crustal rocks reach the Curie temperature. The Curie depth can be used to constrain deep geothermal gradients.

- For pure magnetite the Curie temperature of 580 °C; however, inclusions of titanium can reduce that temperature to values as low as 300 °C.
- In andesites and alkali-basalts ferromagnetic minerals have Curie temperatures ranging from 100 to 300 °C.
- Intermediate to mafic rocks have Curie temperatures ranging from 300 to 450 °C.
- Fe-Co-Ni alloys have Curie temperatures ranging from 620 to 1100 °C.

### Curie Depth

Several authors have used the theoretical possibility to determine Curie depths from total magnetic field intensity data. The area covered must be about 200 x 200 km, with a maximum grid spacing of 1 km. Curie depth estimates are based on a well known method developed by Spector and Grant.

The Curie depth does not necessarily define an isotherm. As a matter of fact, different rock types have different Curie temperatures, which means that the Curie depth can also correspond to a composition boundary.

#### Curie Depth

Seismic or gravity data may be used to infer if the Curie depth corresponds to an isotherm or to composition boundary. When the Curie depth coincides with an inferred velocity boundary or an inferred density boundary, it is probable it reflects a composition boundary. When the Curie depth does not coincide with a velocity boundary or density boundary, it is probable it represents the Curie temperature isotherm (about 580 °C in most continental areas).

The Curie depth-temperature pair then allows the to estimate the deep geothermal gradient in the study area.

#### **Xenoliths**

By definition a xenolith is a piece of country rock picked up by magma as it rises through the crust. The pressuretemperature stability conditions indicated by mineral assemblage in the a xenolith provide information on the depth and temperature of its origin point. Thus, xenoliths provide an independent estimate of temperature at depths down to hundreds of kilometres (e.g. O'Reilly and Griffm, 1985).

# **Upper Mantle Resistivity**

The electrical resistivity of mantle rocks is strongly dependent on temperature. So, in principle, electrical resistivity can be used to infer temperature in the lower crust and upper mantle.

Some authors have identified a direct relationship between the 450 °C isotherm and the depth to an electrically conductive layer in the Canadian Cordillera and in other regions of the Earth.

Magnetotelluric methods are used to make the electrical resistivity measurements and can provide information about the depth of the 450 °C isotherm.

However, the depth resolution is poor.

# Surface temperature (onshore)

When temperatures are measured in boreholes to estimate the geothermal gradient it is also important to know the temperature at the surface of the Earth; that is a way of constraining the geothermal gradient for stationary conditions.

Onshore the temperature of the surface rocks generally exceeds the average air temperature by a few degrees due to surface albedo and interaction of the solar radiation with the ground.

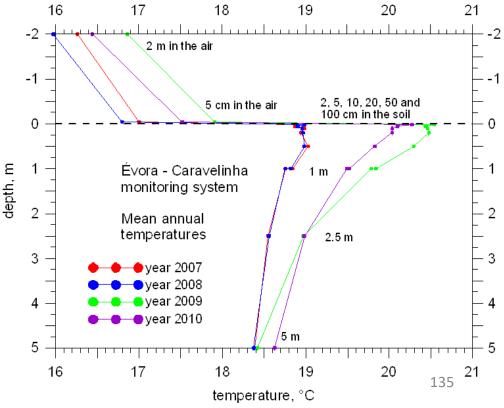
The best way to estimate the average surface temperature  $(T_o)$  is to use local meteorological records and the equation:

$$T_0 = 3 + \frac{(T_{av.min} + T_{av.max})}{2}$$

# Surface temperature (onshore)



$$T_0 = 3 + \frac{(T_{av.min} + T_{av.max})}{2}$$

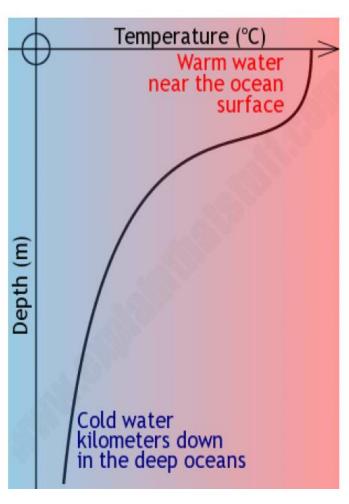


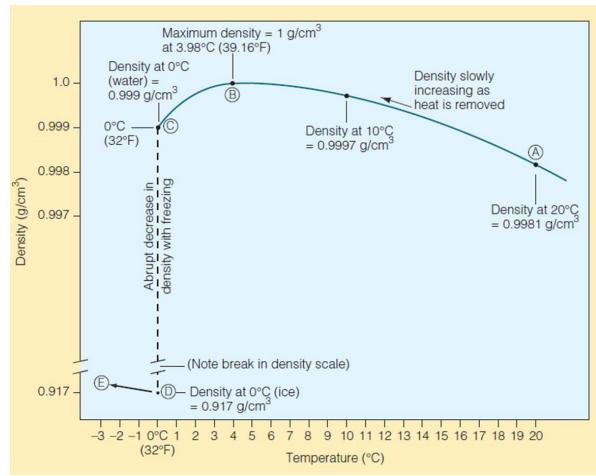
# Surface temperature (offshore)

When speaking about "surface temperature" for offshore temperature logs people is referring to the top of the sediment column or the bottom of the water column.

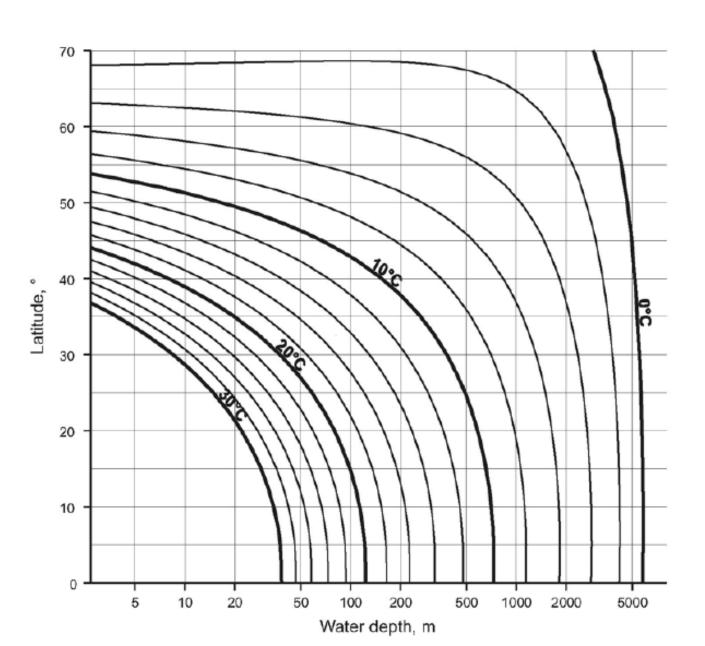
In this case the surface temperature is the bottom water temperature which is obviously different from the average sea surface temperature. Therefore, to calculate a geothermal gradient we must know the temperature at the bottom of the water column, in case we are looking at a conductive process of heat transfer. For the ocean there are graphs that give the temperature at their bottom in accordance with some ocean models.

# Surface temperature (offshore)



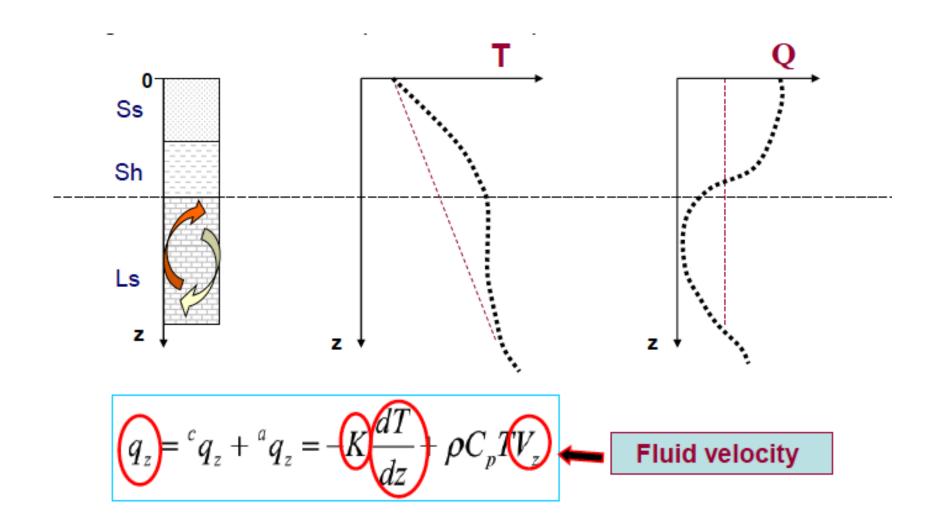


# Surface temperature (offshore)



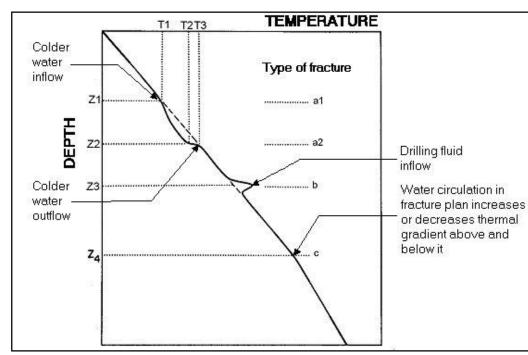
Bottom-water temperature (BWT) (°C) as a function of water depth (m) and latitude (°)

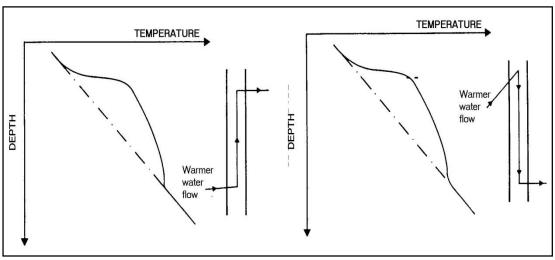
#### Fluid flow in boreholes



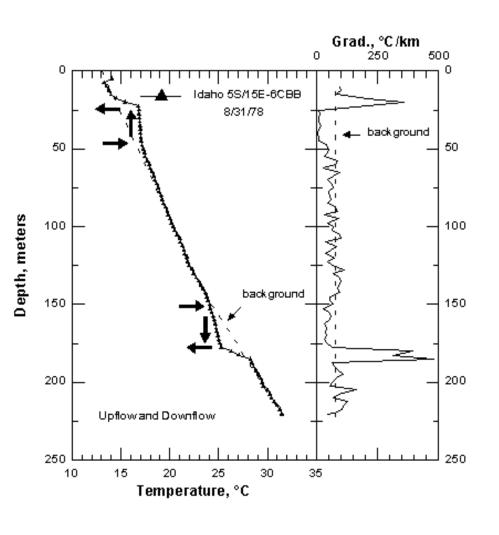
### Fluid flow in boreholes

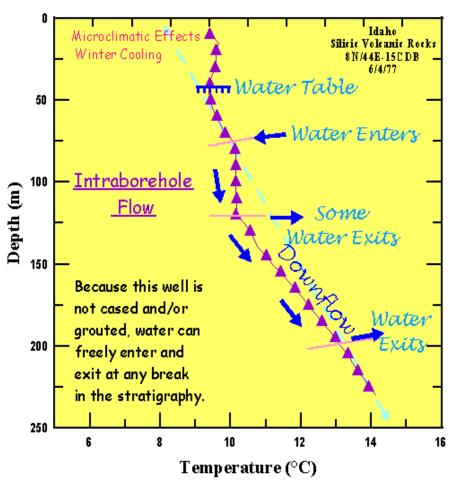
Temperature logs with evidence of fluid flow





# Fluid flow in boreholes





#### Direct temperature measurements

**Bottom-Hole Temperatures (BHT)** 

Drilling in the oil industry consist of (very short summary):

- Start to drill to, let's say, 200 m using mud to cool the drill bit and transport the cuttings to the surface
- Stop drilling and start mud circulation for some time (let's say 2) hours) to homogenize the mud column for well logging
- Pull out the drilling column
- Carry out several runs with different probes for measuring, e.g., gamma ray, electrical resistivity, spontaneous potential, density logs, etc.

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#### Direct temperature measurements

**Bottom-Hole Temperatures (BHT)** 

Drilling in the oil industry consist of (very short summary (cont.)):

- If we are lucky the operators of the geophysical probes will attach a maximum thermometer to each probe
- Each probe reaches the bottom of the borehole some time after the mud circulation has stopped and records the temperature (BHT); the time each probe reaches the bottom is also registered
- Well logging comes to an end and drilling restarts to a new depth (let's say 400 m) and the drilling, the mud circulation and the well logging processes repeat to the new depth

#### Direct temperature measurements

#### Bottom-Hole Temperatures (BHT)

So, for each depth (200 m, 400 m, and so on) we have:

- Several BHT (we must have at least 2)
- The time each probe reached the bottom of the borehole (200 m, 400 m, and so on) after circulation has stopped
- And the circulation time
- With this information it is possible to correct the BHT to obtain the virgin temperature of the geological formations crossed by the borehole
- Sometimes, though, the logging companies also carry out a continuous temperature log, which also has to be corrected for thermal disturbances during the drilling.

# III Thermal Conductivity

#### Reminder

$$q_z = -\frac{\Delta Q}{A \cdot \Delta t} = -K \frac{dT}{dz}$$
  $\alpha = \frac{K}{\rho \cdot c_p}$ 

 $q_z$  is a vector as well as the temperature gradient dT/dz. To calculate (estimate) the heat flux we have to know thermal conductivity K and geothermal gradient dT/dz. Most of the times a  $q_z$  vertical is assumed.  $\alpha$  is the thermal diffusivity.

## Thermo-physical properties

$$\alpha = \frac{K}{\rho \cdot c_p}$$

Thermal conductivity is a measure of how easily heat is transmitted through a material by conduction (W/mK)

**Thermal diffusivity** is a measure of how easily a given material accumulates or looses heat. It controls the rate at which heat dissipates through that material (m<sup>2</sup>/s)

The **specific heat** is the amount of thermal energy that has to be supplied to the unit mass of a given material to raise its temperature 1 K (or the amount of thermal energy that has to be withdrawn from the unit mass of a given material to decrease its temperature 1 K) (J/kgK)

Table 4.1. Thermal Conductivity,  $\lambda$ , of Some Common Minerals

Mineral	Temp. range (K)	$\lambda \ (W  m^{-1}  K^{-1})$	Comments
Quartz	273.2-623.2	$4134 \times T^{-1.053}$	Parallel to c-axis <sup>1</sup>
	273.2-623.2	$820 \times T^{-0.861}$	Perpendicular to c-axis <sup>1</sup>
		7.69	Average <sup>2</sup>
Calcite	273.2-374.2	$264.5 \times T^{-0.727}$	Perpendicular to c-axis <sup>1</sup>
	273.2	5.51	Parallel to $c$ -axis <sup>1</sup>
	332	5.16	Average <sup>1</sup>
		3.59	Average <sup>2</sup>
Dolomite		5.51	Average <sup>2</sup>
Orthoclase		2.32	Average <sup>2</sup>
Albite		2.37	Average <sup>2</sup>
Anorthite		1.68	Average <sup>2</sup>
Plagioclase		1.91	Average <sup>2</sup>
Sillimanite	333.2	2.60	Average <sup>1</sup>
Cordierite	320.8-398.1	$116.3 \times T^{-0.635}$	Average <sup>1</sup>
Salt	273-460	$4610 \times T^{-1.146}$	Average <sup>1</sup>
Topaz	314.2-419.8	$9946 \times T^{-1.094}$	Average <sup>1</sup>
Forsterite		5.12	Average <sup>1</sup>
Wollastonite	317.2-397.2	2.65	Average <sup>1</sup>
Zircon	318.8-411.6	4.03	Parallel to c-axis <sup>1</sup>
	318.7-414.2	4.14	Perpendicular to c-axis <sup>1</sup>
Tourmaline	398.2-723.2	$0.492 \times T^{0.297}$	Parallel to c-axis <sup>1</sup>
	393.2-729.2	$0.108 \times T^{0.556}$	Perpendicular to c-axis <sup>1</sup>

Sources: 1 Touloukian et al. (1970b), 2 Horai and Simmons (1969).

Table 4.2. Comparison of Published Compilations of Thermal Conductivities (W m<sup>-1</sup> K<sup>-1</sup>)

		Sources									
Lithology	$1^a$	2	3	4	5	6	7	8	9	10	11
Sandstone	7.1	$4.2 \pm 1.4$	$3.1 \pm 1.3$		$3.7 \pm 1.2$	2.8		$3.7 \pm 1.2$			$4.7 \pm 2.8$
Claystone	2.9				2.0						1.8
Mudstone	2.9							$2.0 \pm 0.4$			$1.9 \pm 0.4$
Shale	2.9	$1.5 \pm 0.5$	$1.4 \pm 0.4$		$2.1 \pm 0.4$	1.4		$2.1 \pm 0.4$			$1.8 \pm 1.2$
Kaolinite									$1.8 \pm 0.3$		
Glauconite									$0.5 \pm 0.2$		
Siltstone	2.9	$2.7 \pm 0.9$	$3.2 \pm 1.3$		$2.7 \pm 0.2$	$2.7 \pm 0.9$		$2.7 \pm 0.2$			
Limestone	3.1	$2.9 \pm 0.9$	$2.4 \pm 0.9$	2.21	$2.8 \pm 0.4$		$3.4 \pm 3.0$	$2.8 \pm 0.3$			$2.5 \pm 0.6$
Marl	3.2	$2.1 \pm 0.7$	$3.0 \pm 1.1$		$2.7 \pm 0.5$						$2.4 \pm 0.5$
Dolomite		$5.0 \pm 0.6$	$3.1\pm1.4$		$4.7 \pm 0.8$		$4.8\pm1.5$	$4.7 \pm 1.1$			$3.7 \pm 1.8$
Halite		$5.5 \pm 1.8$	$5.7 \pm 1.0$		$5.4 \pm 1.0$			$5.4 \pm 0.3$			5.9
Chert		$4.2 \pm 1.5$	$1.4 \pm 0.5$		$1.4 \pm 0.5$						
Quartzite				6.0			$5.0 \pm 2.4$	$5.9 \pm 0.8$		$3.5 \pm 0.4$	$5.6 \pm 1.9$
Granite							$3.4 \pm 1.2$			$3.5 \pm 0.4$	$2.8 \pm 0.6$
Basalt	1.8			1.7			$1.7 \pm 0.6$			$2.0 \pm 0.2$	1.5
Tuff				$1.7\pm0.3$							
Conglomerate		$2.4 \pm 0.8$	$3.2 \pm 1.8$		$2.1\pm1.0$						
Coal		$0.3 \pm 0.1$	$0.2 \pm 0.2$	$0.2\pm0.04$	$0.2 \pm 0.1$	$0.3 \pm 0.1$					
Loose sand				$2.44 \pm 0.8$							
Typical sediment			$2.3\pm2.0$								

<sup>&</sup>lt;sup>a</sup>Matrix conductivity values, only representing bulk conductivity when  $\phi = 0$ .

Sources: 1 = Beardsmore (1996), 2 = Majorowicz and Jessop (1981), 3 = Beach, Jones and Majorowicz (1987), 4 = Raznjevic (1976), 5 = Reiter and Jessop (1985), 6 = Taylor, Judge and Allen (1986), 7 = Roy et al. (1981), 8 = Reiter and Tovar (1982), 9 = Touloukian et al. (1970b), 10 = Drury (1986), 11 = Barker (1996).

Table 4.3. Thermal Conductivity of Common Pore Fluids

Fluid	Conductivity (W m <sup>-1</sup> K <sup>-1</sup> )	Source
Water		
Fresh	$-7.42 \times 10^{-6} \times T^2 + 5.99 \times 10^{-3} \times T - 0.522$	1
Saline	$-7.42 \times 10^{-6} \times T^2 + 5.99 \times 10^{-3} \times T - 0.5$	
Hydrocarbon		
Oil	0.1	1
Gas	$0.000143 \times T - 0.0159$	1
Air	0.023	2

*Note*: T = absolute temperature in the range 295–450 K.

Source: 1 = Touloukian et al. (1970a), 2 = Giancoli (1984).

## Thermal conductivity and lithology

Thermal conductivity is closely related to lithology. Whenever possible, each lithology within the region of interest should be sampled for thermal conductivity measurements. Ideally, these should use a steady-state method on core samples, but drill cuttings and transient methods can also be used with reduced accuracy and precision.

The important parameter to be determined is thermal conductivity of the matrix of the rock.

Apart from temperature, thermal conductivity also varies with pressure, saturation, pore fluid, dominant mineral phase, and anisotropy of different rock types.

## Thermal conductivity and compaction

Compaction reduces the amount of pore fluid in a rock and generally increases the bulk thermal conductivity.

Different lithologies compact at different rates.

In the absence of porosity logs, compaction models must be constructed for each lithology under investigation.

Once porosity is known, it can be combined with matrix thermal conductivity to determine the bulk thermal conductivity of each formation. This thermal conductivity must then be corrected for in situ temperature conditions.

## Thermal conductivity measurement

Thermal conductivity is measured on samples from wells (cores or large cuttings or cell with small cuttings) or in samples from rock outcrops.

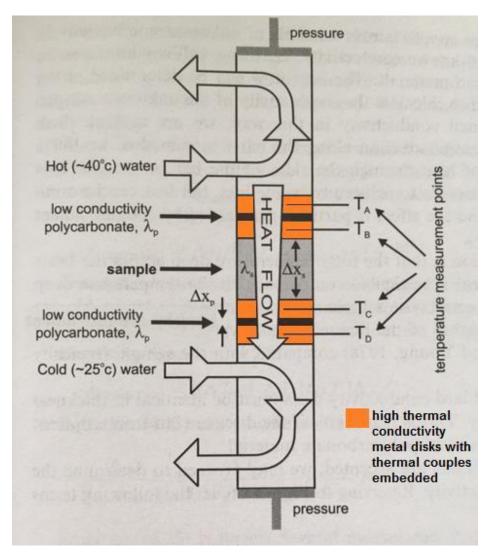
There are two main methods to measure thermal conductivity:

- "Divided Bar" measures K in steady state mode
- Transient methods, with a linear source of heat (needle within the sample) or a planar source of heat (hot plate on the sample surface) in transient state

## Steady-State Methods

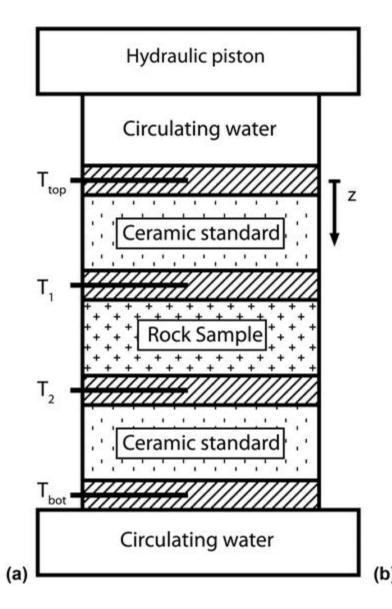
Consolidated rocks

## Divided-bar apparatus





## Divided-bar apparatus





### Divided-bar apparatus

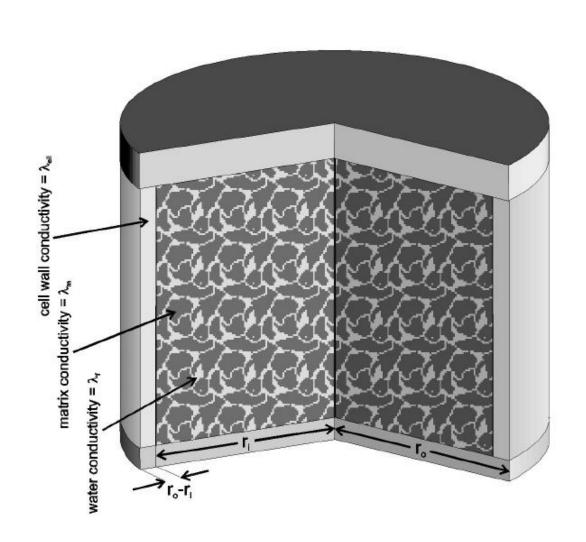
Sample preparation: they should be cut as cylinders with the bases polished and saturated with water

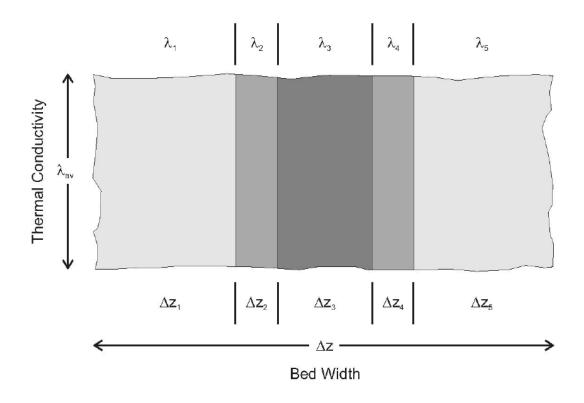
The heat flow density is constant along the pile

$$Q = K_s \cdot \frac{\Delta T_s}{\Delta x_s} = K_r \cdot \frac{\Delta T_r}{\Delta x_r}$$

Solving for the thermal conductivity of the rock sample we get

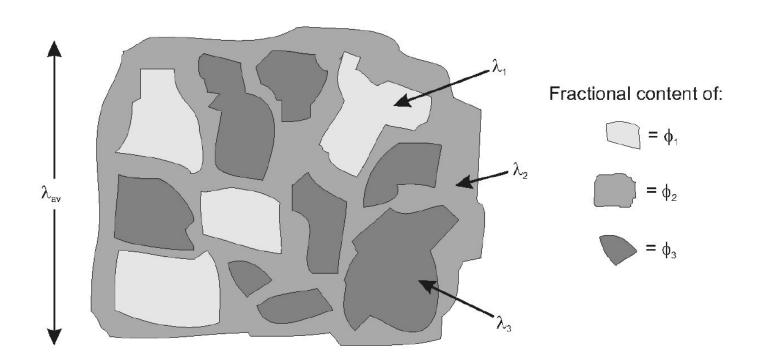
$$K_{r} = K_{s} \cdot \frac{\Delta T_{s}}{\Delta x_{s}} \cdot \frac{\Delta x_{r}}{\Delta T_{r}}$$





Heat flow parallel to the bedding: apply the aritmetic mean

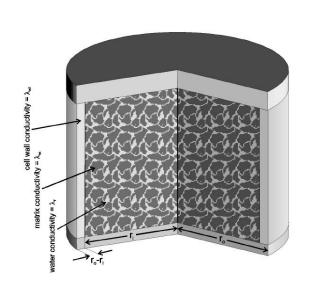
$$\lambda_{total} = \frac{\sum_{i=1}^{n} (\Delta z_i \cdot \lambda_i)}{\Delta z}$$



Square-root mean (Roy, Beck and Touloukian (1981))

$$\sqrt{\lambda_{\mathrm{B}}} = \sum_{\mathrm{i=1}}^{\mathrm{n}} \phi_{\mathrm{i}} \cdot \sqrt{\lambda_{\mathrm{i}}}$$

$$\lambda_{total} = \lambda_{ag} \cdot \frac{\pi(r_i)^2}{\pi(r_0)^2} + \lambda_{cell} \cdot \frac{\pi(r_0)^2 - \pi(r_i)^2}{\pi(r_0)^2}$$



$$\lambda_{\text{ag}} = (\lambda_{total} - \lambda_{\text{cell}}) \left(\frac{r_0}{r_i}\right) + \lambda_{\text{cell}}$$

$$\lambda_{\rm m} = \left[ \frac{\sqrt{\lambda_{\rm ag}} - \emptyset \sqrt{\lambda_{\rm w}}}{(1 - \emptyset)} \right]^2$$

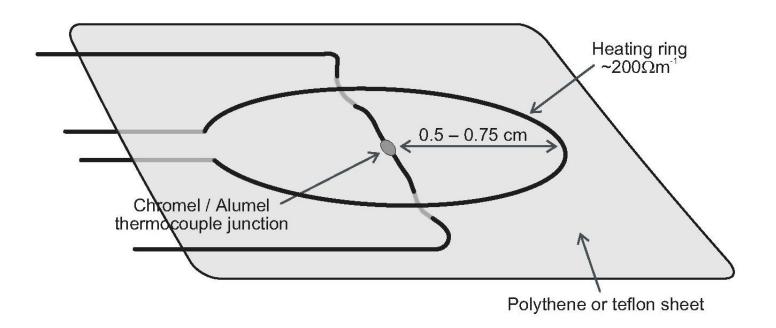
#### Transient methods

Unconsolidated rocks

## Thermal conductivity meter

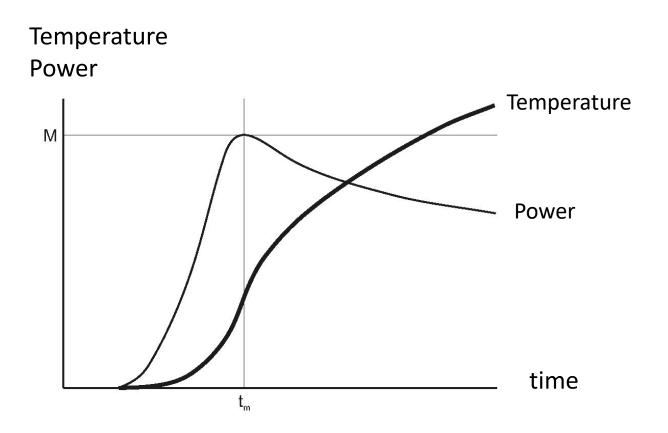


#### Line-source methods



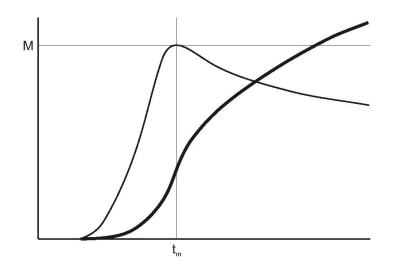
$$K = 0.0771 \cdot \frac{Q}{M \cdot t_m}$$

#### Line-source methods



Temperature vs time (thick line) and heating rate vs time (thin line) at a position offset from the line heat source

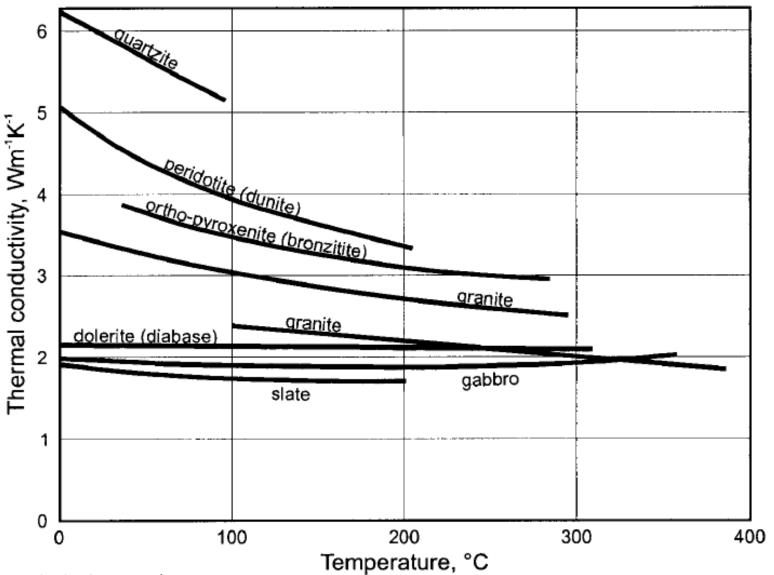
#### Line-source methods



$$K = 0.0771 \cdot \frac{Q}{M \cdot t_{m}}$$

where Q (Wm<sup>-1</sup>) is the heat liberated per unit length of wire per unit time which is equal to  $Q_T/2\pi r$  and  $Q_T$  (W) is the total amount of heat produced per unit time by a ring of radius r.

### Thermal conductivity vs. temperature



(Birch and Clark, 1940).

### Thermal conductivity vs. temperature

$$K(T) = [K(22)] \cdot \left(\frac{295}{T + 273}\right)$$

Correia et al., 1989

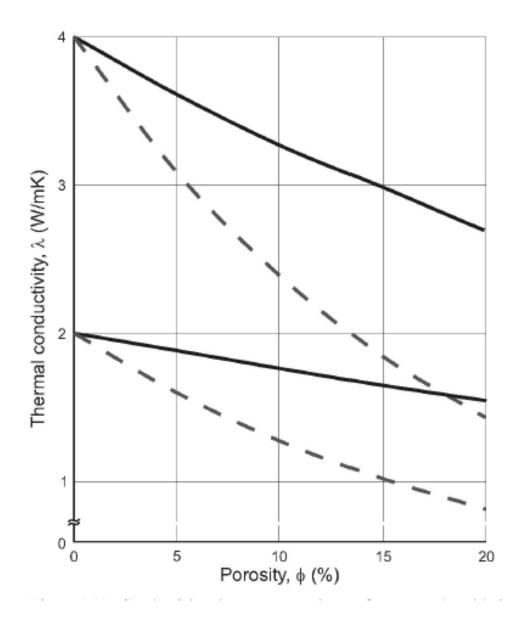
$$\lambda = A + \frac{B}{350 + T}$$

Zoth and Haenel, 1988

Table 8.13. Values for the constants A and B in eq. (8.33) for different rock types; data: [1988Zot].

rock type	T (°C)	A (W m <sup>-1</sup> K <sup>-1</sup> )	B (W m <sup>-1</sup> )
(1) rock salt	-20 – 0	-2.11	2960
(2) limestones	0 – 500	0.13	1073
(3) metamorphic rocks	0 - 1200	0.75	705
(4) acid rocks	0 - 1400	0.64	807
(5) basic rocks	50 - 1100	1.18	474
(6) ultra-basic rocks	20 - 1400	0.73	1293
(7) rock types (2)-(5)	0 - 800	0.70	770

## Thermal conductivity vs. porosity



Conductivity versus porosity for two rocks with  $\lambda = 2$  W/mK and with  $\lambda = 4$  W/mK. Solid line assumes water-filled pores, dashed line air-filled pores. Modified after Jessop (1990).

### Thermal conductivity vs. porosity

There are at least three popular models describing the compaction of sediments with increasing burial.

That of Sclater and Christie (1980) who state that porosity decays exponentially with depth of burial, z:

$$\emptyset = \emptyset_0 \cdot e^{(-Az)}$$

where  $\emptyset_0$  is the porosity of the sediments at the time of deposition and A is constant called compaction coefficient (A $^2$ .7 x 10<sup>-4</sup>).

That of Falvey and Middleton (1981) who proposed:

$$\frac{1}{\emptyset} = \frac{1}{\emptyset_0} + Bz$$

where B is a constant compaction coefficient ( $B^7.8688 \times 10^{-4}$ ).

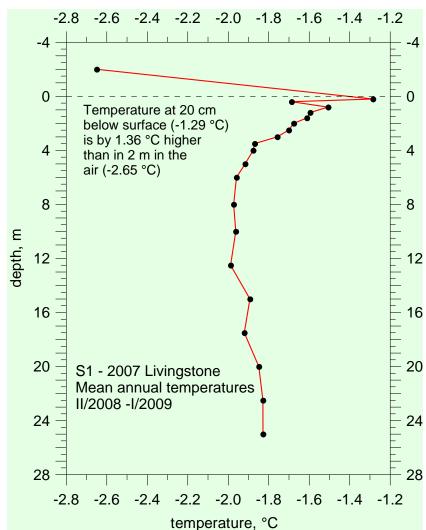
## Thermal conductivity vs porosity

And that of Baldwin and Butler (1985) who suggested that while the Sclater and Christie (1980) model is best for sandstone, the compaction of shale and limestone is best explained using a power law model:

$$z = z_{max}(1 - \emptyset)^{C}$$

where  $z_{max}$  is the depth at which all fluid is expelled and C is a compaction constant ( $C^{\sim}6.35$ ).





#### Reminder

$$\frac{\partial \mathbf{T}}{\partial \mathbf{t}} = \alpha \frac{\partial^2 \mathbf{T}}{\partial \mathbf{z}^2}$$

$$T(z = 0, t) = T_0 + A_0 \sin(\omega t - \varepsilon_0)$$

$$T(z,t) = T_0 + z \cdot \operatorname{grad} T + A_0 \cdot e^{-z/d} \cdot \sin(\omega t - z/d - \varepsilon_0)$$

where grad T is the geothermal gradient and  $d=\sqrt{2\alpha/\omega}$ 

$$\alpha = \frac{\omega \cdot (z_2 - z_1)^2}{2 \cdot \left( \ln(A_1/A_2) \right)^2}$$

## Periodic temperature change at the Earth's surface

The heat conduction equation can be integrated to give the temperature distribution as a function of time (t) and depth (z)

$$\frac{\partial \mathbf{T}}{\partial \mathbf{t}} = \alpha \cdot \frac{\partial^2 T}{\partial z^2}$$

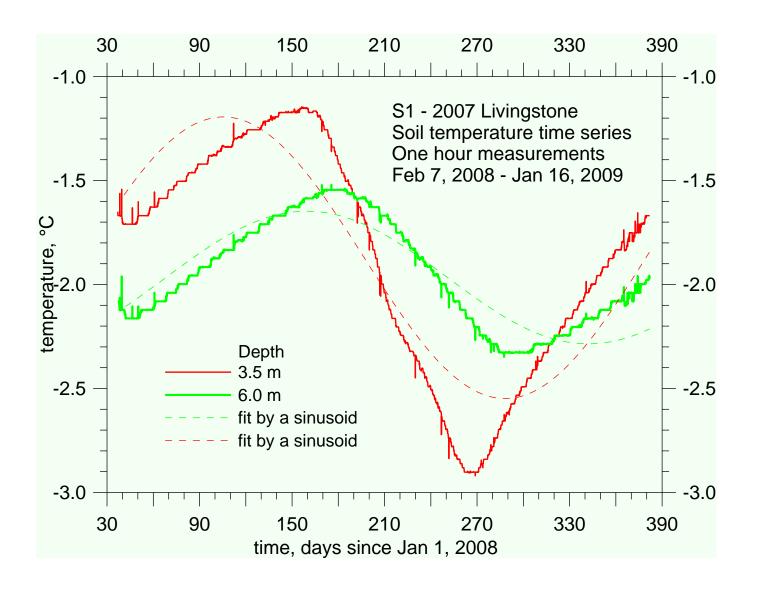
$$T(0,t) = T_0 \cdot e^{i\omega t}$$

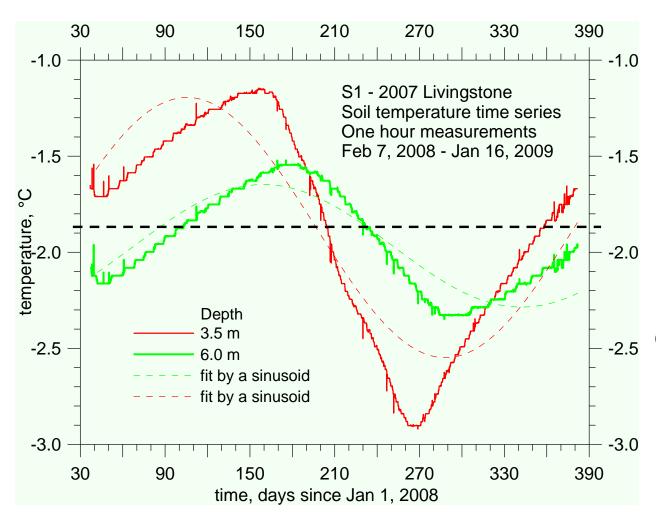
$$T(z,t) \to 0 \quad \text{for} \quad z \to \infty$$

$$T(z,t) = T_{0} \cdot exp \left( -\sqrt{\frac{\omega \cdot \rho \cdot c_{p}}{2K}} \cdot z \right) \cdot exp \left[ i \left( \omega \cdot t - \sqrt{\frac{\omega \cdot \rho \cdot c_{p}}{2K}} \cdot z \right) \right]$$

$$L = \sqrt{\frac{2K}{\omega \cdot \rho \cdot c_{p}}}$$

$$\Phi = \sqrt{\frac{\omega \cdot \rho \cdot c_{p}}{2K}}$$





$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2}$$

$$\alpha = \frac{\omega \cdot (z_2 - z_1)^2}{2 \cdot \left(\ln(A_1/A_2)\right)^2}$$

#### A few results

- The thermal conductivity in Antarctica samples is similar to those measured in other regions of the Earth.
- The thermal diffusivity measured in Antarctic cores collected in the borehole GP1 range between  $1.09 \times 10^{-6}$  and  $1.58 \times 10^{-6}$  m<sup>2</sup>/s.
- The thermal diffusivity estimated using temperatures measured in the borehole is  $2.2 \times 10^{-6} \text{ m}^2/\text{s}$ .

## IV Heat Production

#### Reminder

$$q_z = -\frac{\Delta Q}{A \cdot \Delta t} = -K \frac{dT}{dz}$$

Fourier equation

$$\frac{\partial T}{\partial t} = \alpha \cdot \frac{\partial^2 T}{\partial z^2}$$

Heat conduction equation

$$T = T_0 + \frac{Q_0}{K} \cdot z - \frac{A'}{2K} \cdot z^2$$

## Heat generation

## Radioactivity in the Earth

Heat is generated in rocks principally through the radioactive decay of unstable isotopes that release energy in the form of alpha ( $\alpha$ ) and beta ( $\beta$ ) particles, gamma radiation ( $\Upsilon$ ). However, the surrounding rocks absorb the kinetic energy carried by those particles and radiation, thus generating heat.

The rate of radiogenic heat generation within rocks is related to the abundance of isotopes in them and determines the rate of heat production. Approximately 98% of geothermal radiogenic heat arises from the decay of uranium (U<sup>238</sup>), thorium (Th<sup>232</sup>) and potassium (K<sup>40</sup>). The energy released by the decay of the uranium is considerably greater than the other two.

To determine the heat production of a rock it is necessary to know its uranium, thorium and potassium concentrations which are generally obtained using a gamma-ray spectrometer. Heat production can be calculated using the equation

$$A = \rho \cdot (0.097 \cdot C_{U} + 0.026 \cdot C_{Th} + 0.036 \cdot C_{K})$$

where A is in  $\mu$ W/m<sup>3</sup>,  $\rho$  is the density of the rock (g/cm<sup>3</sup>), and C<sub>U</sub>, C<sub>Th</sub>, and C<sub>K</sub> are the concentrations of U and Th (in ppm) and K (in %), respectively.

	_	oncentration .m. by weight]		Heat production $[10^{-11} \mathrm{W  kg^{-1}}]$			
Rock type	U	Th	K	U	Th	K	Total
Granite	4.6	18	33,000	43.8	46.1	11.5	101
Alkali basalt	0.75	2.5	12,000	7.1	6.4	4.2	18
Tholeiitic basalt	0.11	0.4	1,500	1.05	1.02	0.52	2.6
Peridotite, dunite	0.006	0.02	100	0.057	0.051	0.035	0.14
Chondrites	0.015	0.045	900	0.143	0.115	0.313	0.57
Continental crust	1.2	4.5	15,500	11.4	11.5	5.4	28
Mantle	0.025	0.087	70	0.238	0.223	0.024	0.49

Estimates of radioactive heat production in selected rock types, based on heat production rates (from Rybach, 1976, 1988) and isotopic concentrations.

Rock Type	No. of Samples	$A (\mu \text{W m}^{-3})$	$V_{\rm p}~({\rm kms}^{-1})$	
Acidic rocks				
Rhyolite	5	$2.80 \pm 0.28$	$5.57 \pm 0.12$	
Granite	16	$2.82 \pm 1.03$	$5.85 \pm 0.16$	
Granodiorite	11	$2.45 \pm 1.29$	$5.88 \pm 0.22$	
		1.0		
Tonalite	8	$1.48 \pm 0.54$	$5.89 \pm 0.20$	
Diorite	7	$0.88 \pm 0.30$	$6.24 \pm 0.25$	
Basic rocks				
Gabbro	6	$0.11 \pm 0.13$	$6.19 \pm 0.82$	
		0.03		
Amphibolite	6	$0.37 \pm 0.19$	$6.30 \pm 0.30$	
Ultrabasic rocks				
Hornblendite	3	$0.12 \pm 0.15$	$6.91 \pm 0.78$	
Pyroxinite	4	$0.06 \pm 0.04$	$7.20 \pm 0.33$	
Peridotite	4	$0.01 \pm 0.01$	$7.82 \pm 0.12$	
Serpentinite	6	$0.01 \pm 0.02$	$6.49 \pm 0.64$	
Miscellaneous crustal rocks				
Various	23	$1.91 \pm 1.40$	$5.78 \pm 0.53$	
Granulite		0.13		
Bulk Earth		0.014		
Carbonaceous chondrite		0.01		
Ordinary chondrite		0.015		

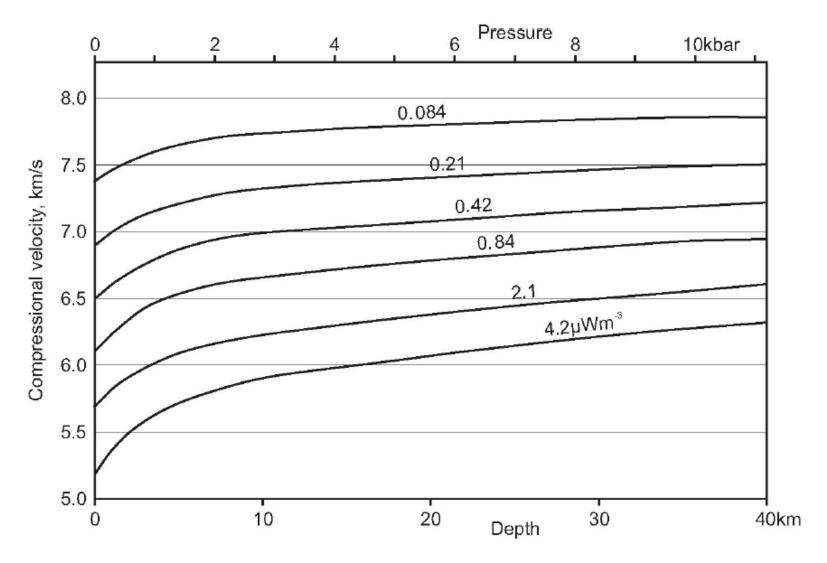
Note: Uncertainty in A and  $V_p$  = one standard deviation.

Sources: Cermák et al. (1990) and Brown and Mussett (1993).

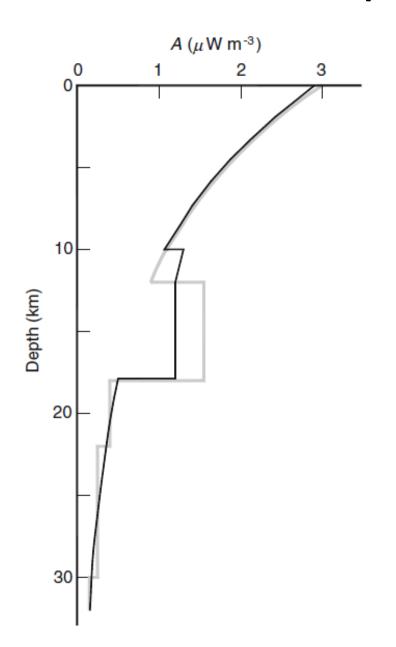
Because many times there no samples to determine the concentrations of U, Th and K, several authors have suggested a relationship between heat production and velocity of P waves (Rybach and Bunterbath, 1984). One of those relationships is

$$ln(A) = B - 2.17 \cdot V_P$$

where A in  $\mu$ W/m³ is the equivalent heat generation for compressional seismic velocity Vp (km/s), and B is equal to 12.6 for Precambrian formations and 13.7 for Phanerozoic formations which reflects variations in crustal evolution and seismic complexity. Some authors have developed other relationships; other authors consider these relationships not realistic.



Correlation between depth/pressure, seismic velocity and heat production. Modified after Rybach (1979)



Radiogenic heat production as a function of depth in the Variscan crust based on a compositional model (grey line) and estimated radiogenic heat production using relationship between heat production and velocity of P waves (black line).

$$ln(A) = B - 2.17 \cdot V_P$$

Experimental results on volcanic rocks show that the radioelement concentration and, consequently, the radiogenic heat are related to magmatic differentiation processes, thus implying a decrease with depth within the crust. The most widely accepted model is the exponential model by Lachenbruch (1970):

$$A(z) = A_0 e^{-\frac{z}{D}}$$

where  $A_o$  (in  $\mu$ W m<sup>-3</sup>) is the radiogenic heat production at the surface and D (in km) is the rate of heat decrease.

Factor D, which ranges from 5 to 15 km, derives from the linear relationship

$$q_0 = q_m + A_0 \cdot D$$

where  $q_o$  is the heat flowing out from the Earth's surface and  $q_m$  is a constant component of heat flow from the mantle. It is widely accepted that the surface radiogenic heat in the continental areas is of the order of 3  $\mu$ W m<sup>-3</sup> as radioactivity measurements of igneous and metamorphic rocks restrict the range of radiogenic heat production to 2.5 – 3.5  $\mu$ W m<sup>-3</sup>.

## Heat flow density

#### Reminder

$$q_z = -\frac{\Delta Q}{A \cdot \Delta t} = -K \frac{dT}{dz}$$

Fourier equation

$$\frac{\partial T}{\partial t} = \alpha \cdot \frac{\partial^2 T}{\partial z^2}$$

Heat conduction equation

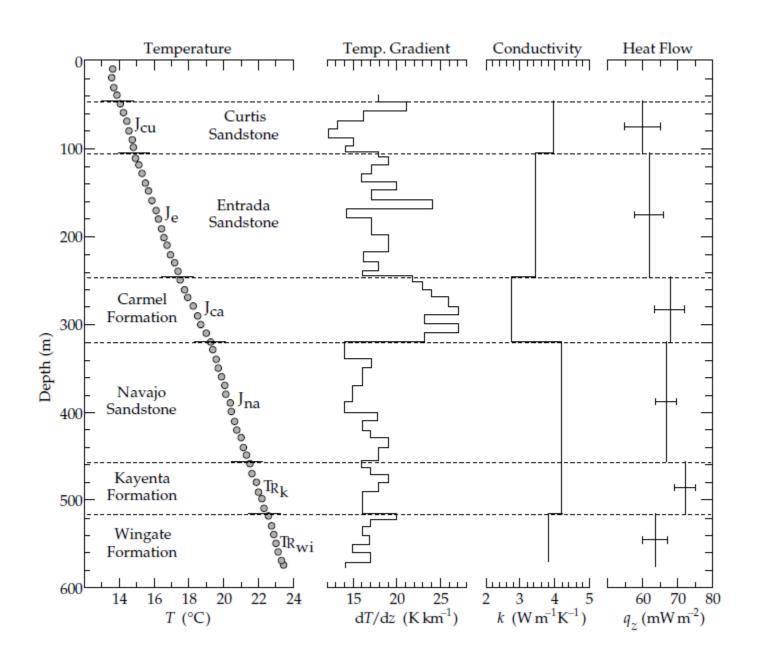
$$T = T_0 + \frac{Q_0}{K} \cdot z - \frac{A'}{2K} \cdot z^2$$

# Heat flow density calculation the product method

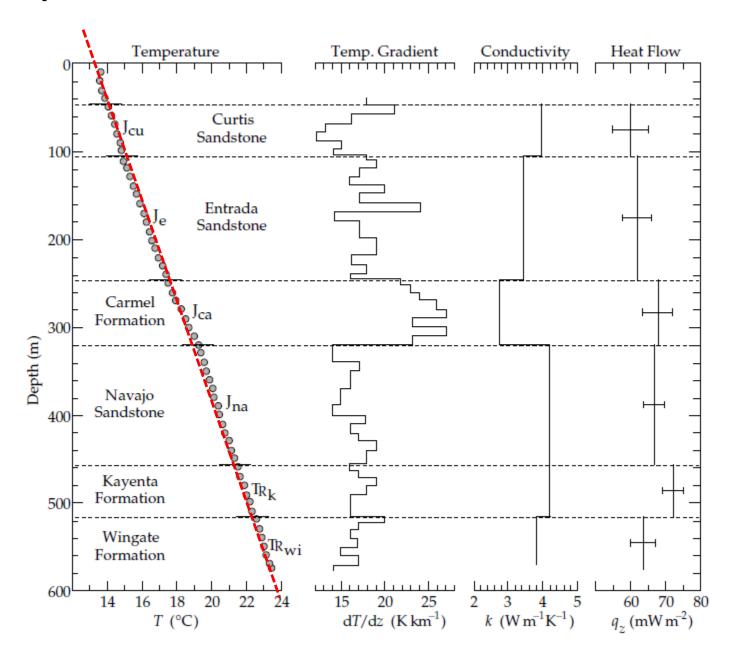
$$q_z = -\frac{\Delta Q}{A \cdot \Delta t} = -K \frac{dT}{dz}$$

To calculate the HFD only linear portions of the temperature log (or, equivalently, pure condutive portions of the temperature log) should be used to multiply by the average thermal conductivity for the same depth interval

#### The product method for HFD calculation



#### The product method for HFD calculation



# Heat flow density calculation the Bullard plot method

A more coherent and demonstrable calculation of heat flow is based on the concept of *thermal resistance*. Thermal resistance (R) is defined as the integral of the reciprocal of thermal conductivity over the depth range z:

$$R = \int \frac{1}{\lambda} dz \qquad \qquad R = \sum_{i=1}^{n} \left( \frac{\Delta z_i}{\lambda_i} \right)$$

In SI units it is expressed in meters squared kelvin per watt (m<sup>2</sup>KW<sup>-1</sup>).

# Heat flow density calculation the Bullard plot method

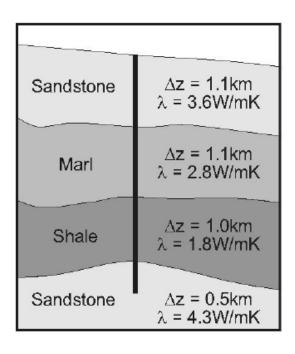
Temperature measured at a shallow level is used to estimate, by extrapolation, the temperature at depth and the heat flow. Assuming a pure conduction (no fluid circulation) the most common equation is (Bullard, 1939):

$$T_{z} = T_{0} + q_{0} \sum_{i=1}^{N} (\Delta_{zi} / k_{i})$$

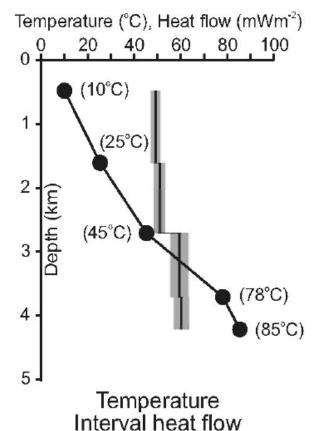
where  $T_z = T$  at depth z,  $T_0 =$  average surface temperature,  $q_0$  i the heat flow density at the surface (constant),  $\Delta_{zi}$  is the thickness of the i<sup>th</sup> unit,  $k_i$  is the thermal conductivity of the i<sup>th</sup> unit. A graph of  $T_z$  vs.  $(\Delta_{zi}/k_i)$  is known as a Bullard plot.

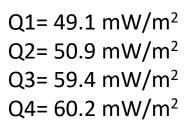
## Product method / Bullard plot method

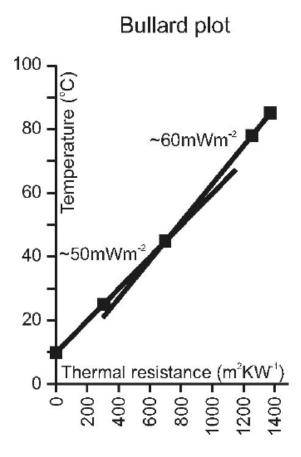
#### Stratigraphy



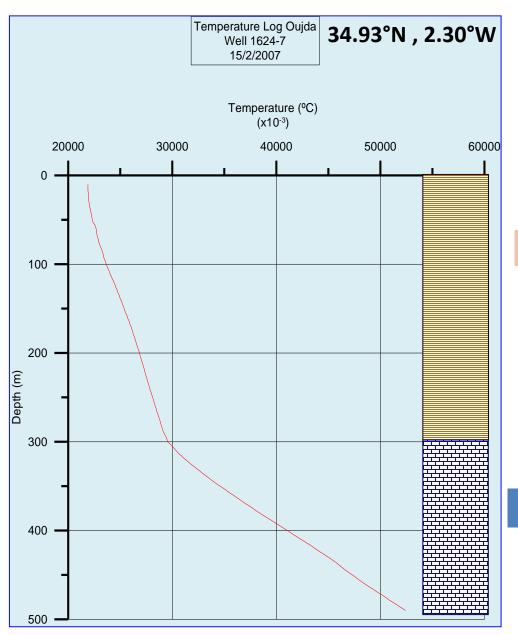
54.7 mW/m<sup>2</sup>







#### Morocco again

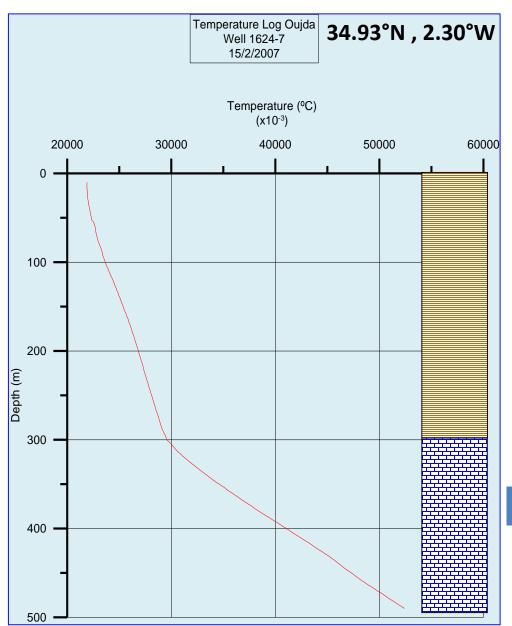


If the clay thermal conductivity were 1.5 W/mK and the Marl/dolomite 4.0 W/mK what would be the heat flow density in each formations? Give a plausible explanation. What would be the temperature at 1000 m depth?

Marl/Dolomite

Clay

#### Morocco again

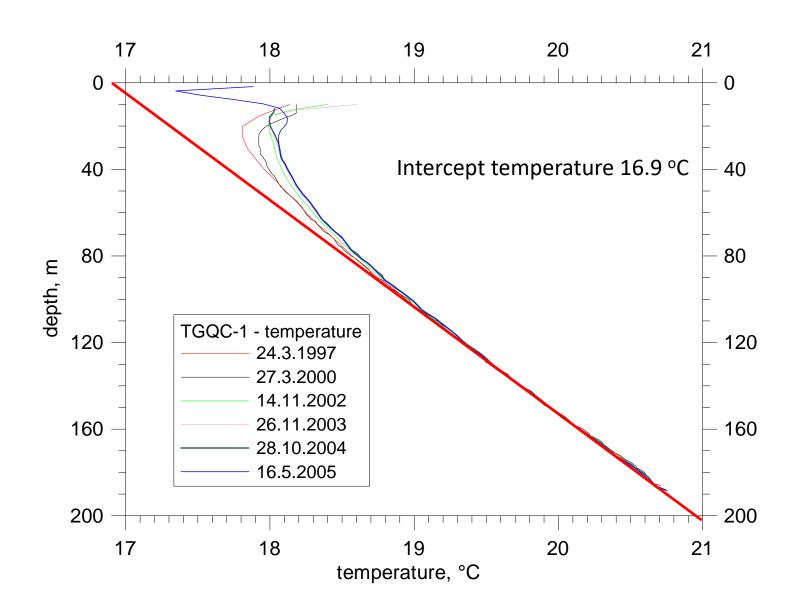


If a thermal conductivity of 1.5 W/mK is assumed for the clay and a thermal conductivity of 4.0 W/mK for the dolomite the heat flow density above 300 m is 43 mW/m<sup>2</sup> while below  $300 \text{ m is } 510 \text{ mW/m}^2.$ Something hot is

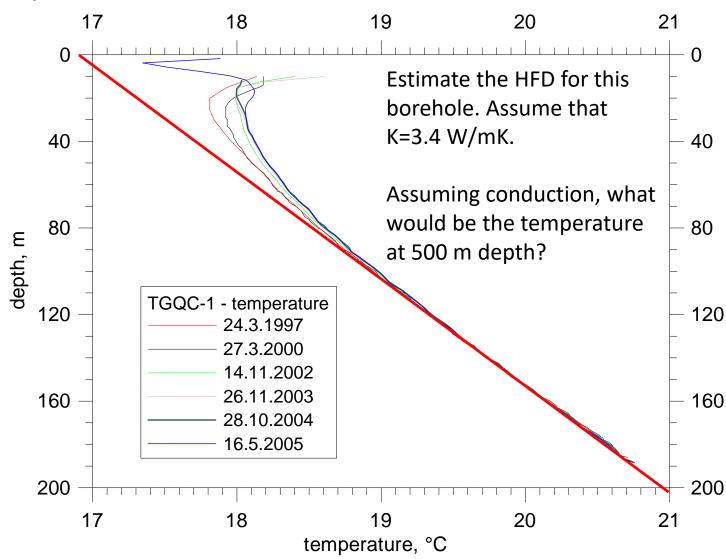
happening here!

Marl/Dolomite

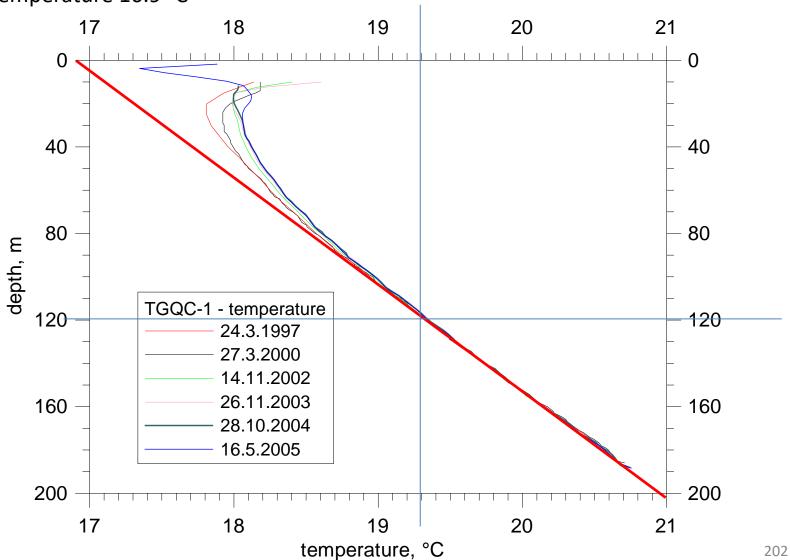
Clay



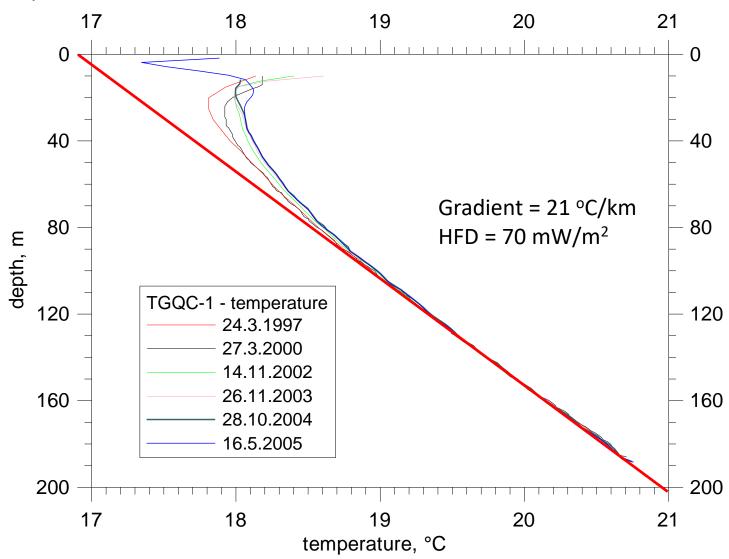
Intercept temperature 16.9 °C



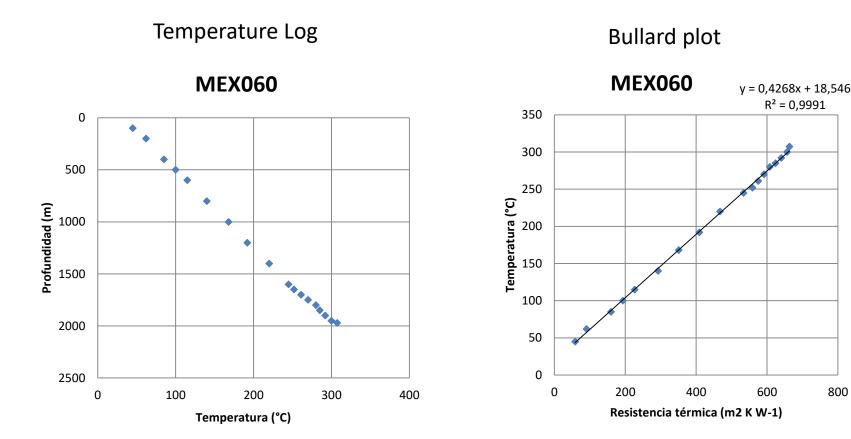
Intercept temperature 16.9 °C



Intercept temperature 16.9 °C



## Mexico - Bullard plot method



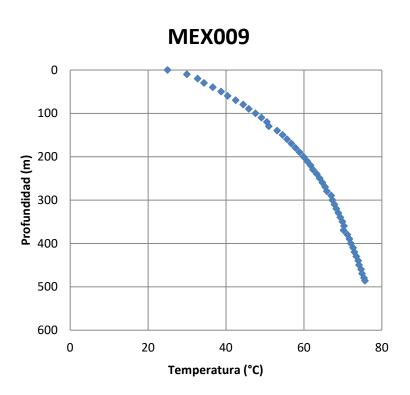
From the graphs estimate the geothermal gradient, the heat flow density, and the average thermal conductivity. 800

 $R^2 = 0,9991$ 

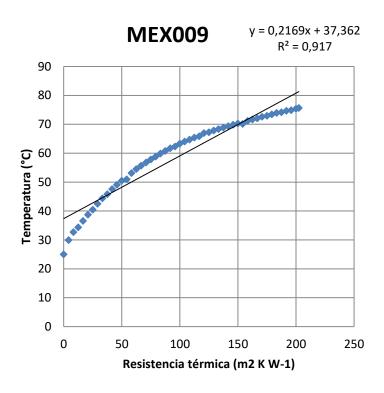
600

## Mexico - Bullard plot method

Temperature Log

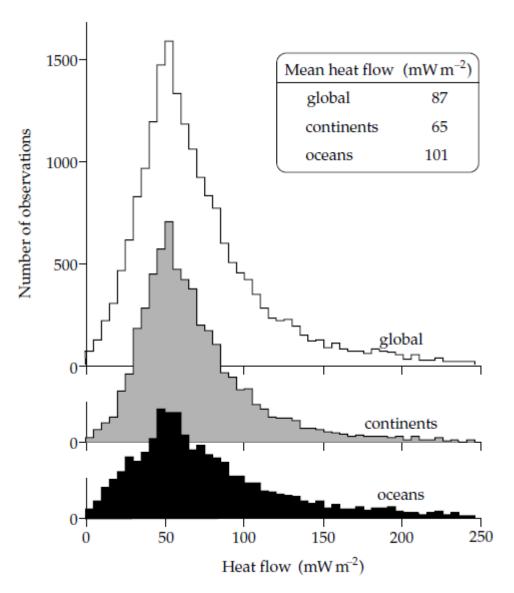


**Bullard plot** 

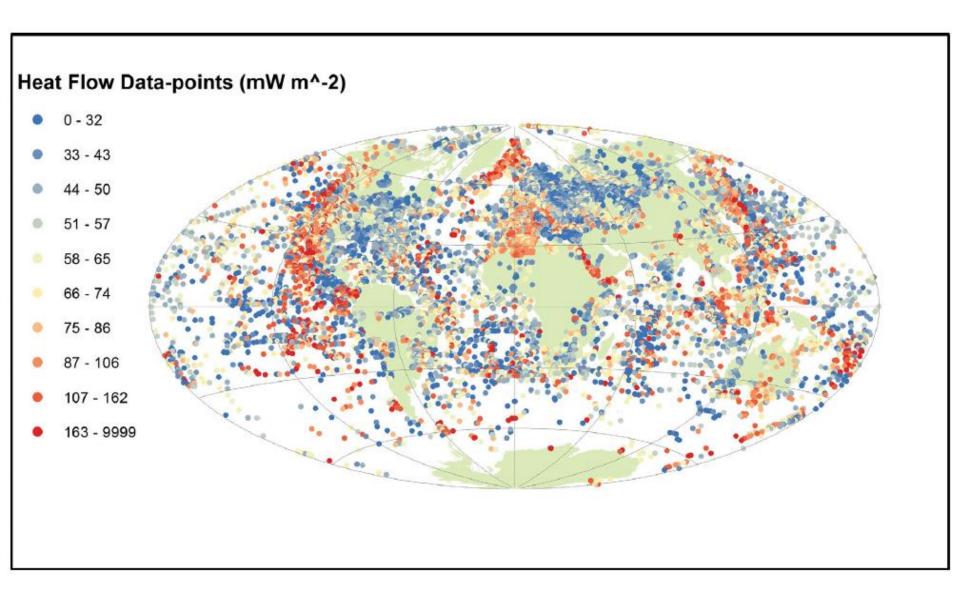


## Global heat flow density

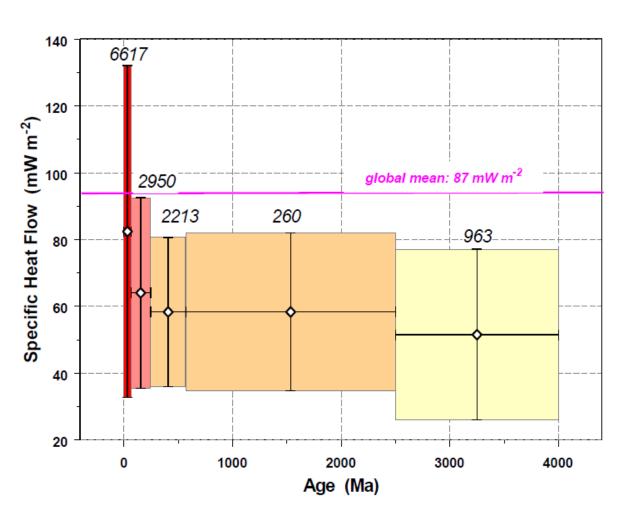
Histograms of continental, oceanic and global heat-flow. Values after Pollack et al. (1993).



## Global heat flow density



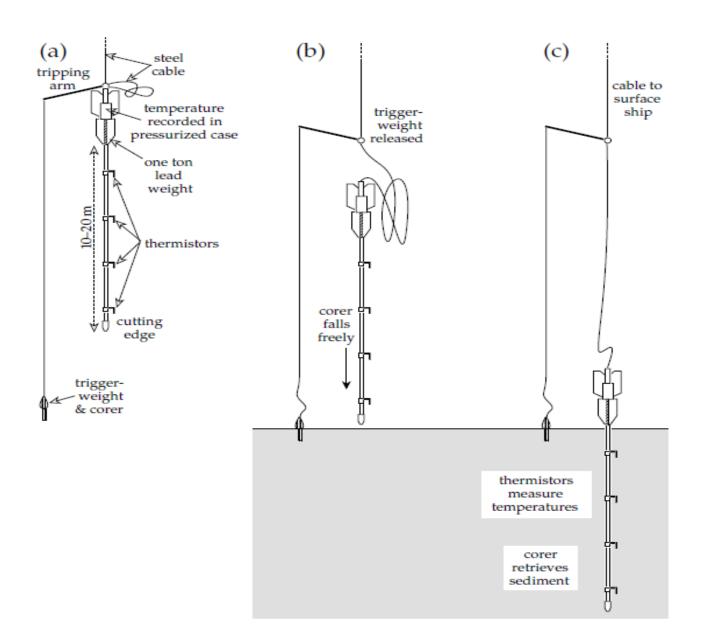
## Global heat flow density

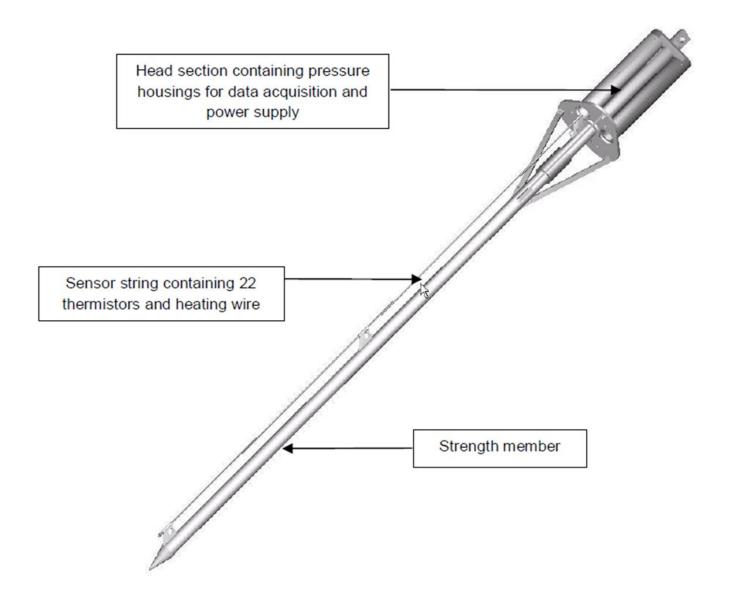


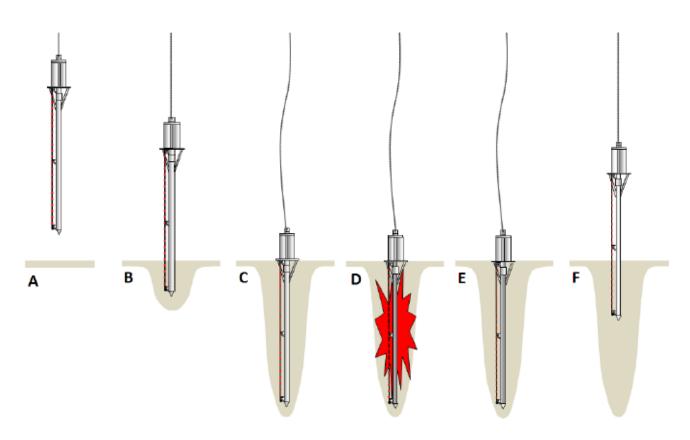
Studies of the heat flow density based on observations at 20201 sites worldwide reveal there is a decrease of the heat flow density with age: heat flow density is lower in old stable platforms than in young, tectonically active crust, on average by a factor of  $1 \frac{1}{2}$ .

As a consequence, the mean heat flow density is larger in the generally young oceans (101 mWm<sup>-2</sup>) than on the continents (67 mWm<sup>-2</sup>).

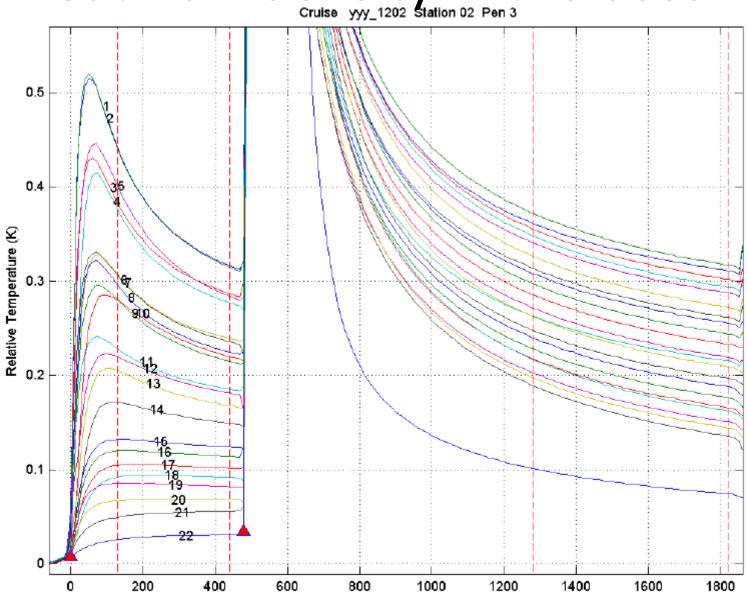




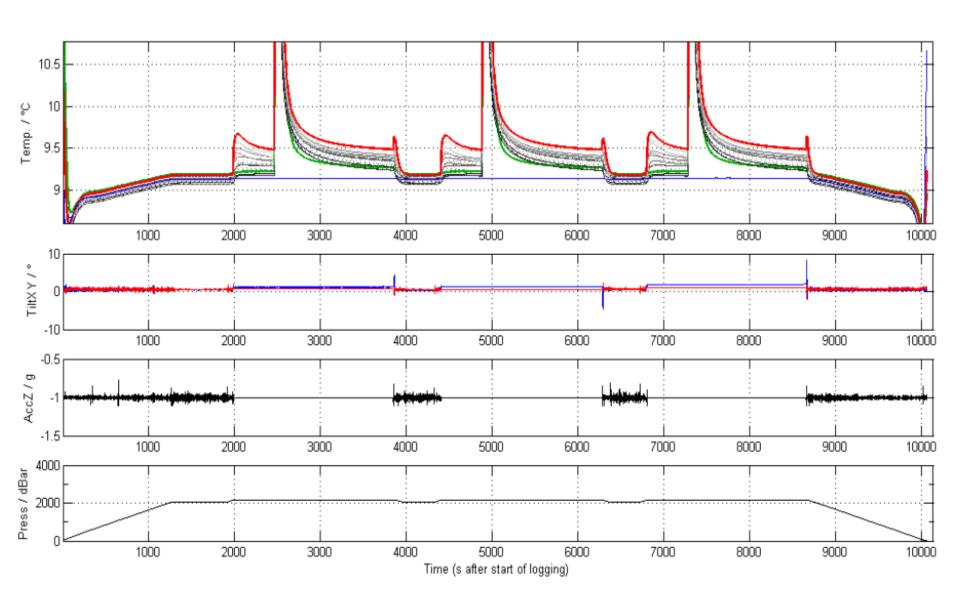


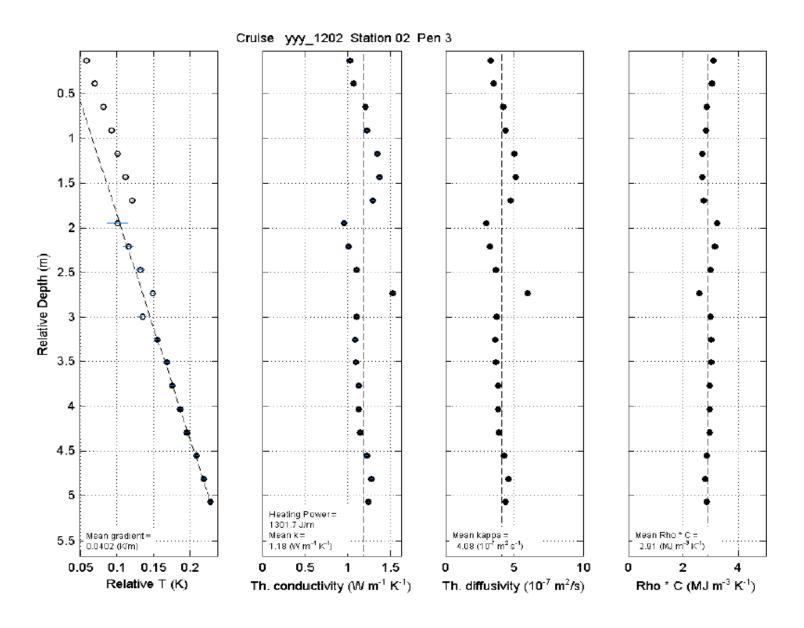


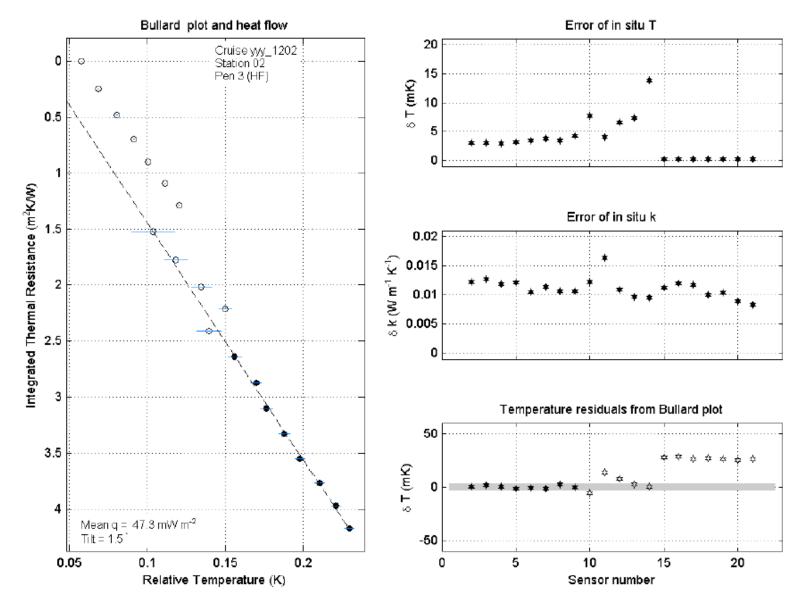
- A: lowering to seabed
- B: penetrating into seabed
- C: measuring thermal decay of frictional heat (approx.. 7-12 minutes)
- D: heat pulse (approx.. 20 seconds)
- E: measuring thermal decay of heat pulse (approx.. 15-20 minutes)
- F: pullout and retrieve to surface



Time (s)



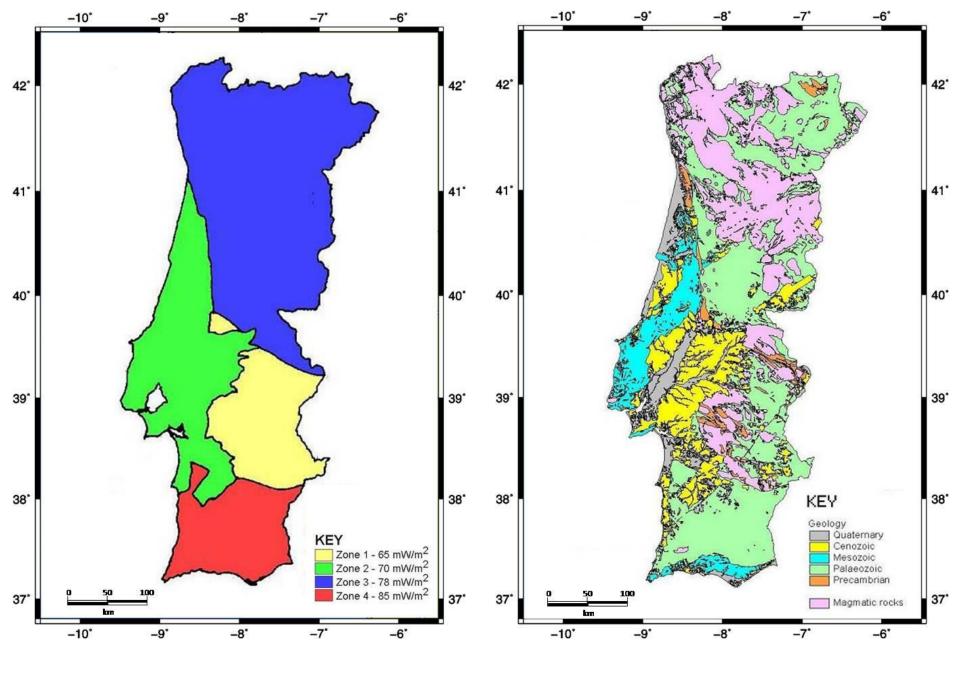


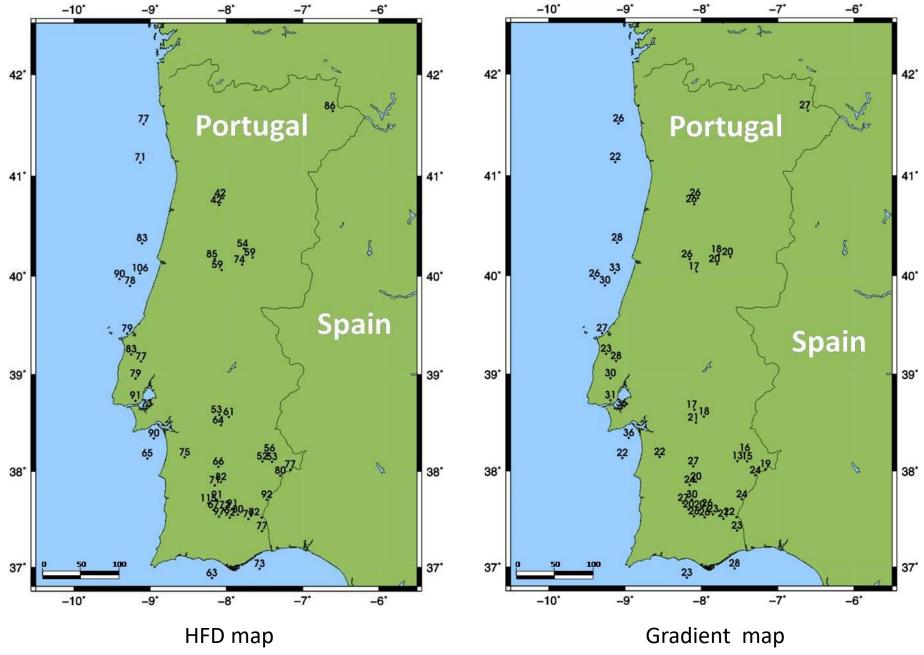


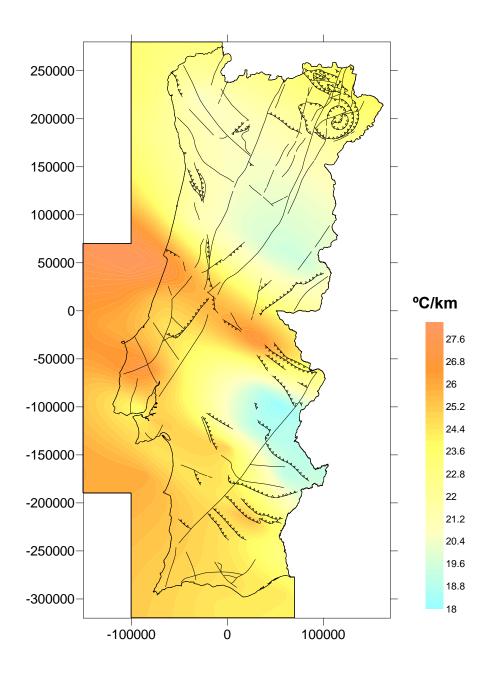




## VI Geothermal Resources



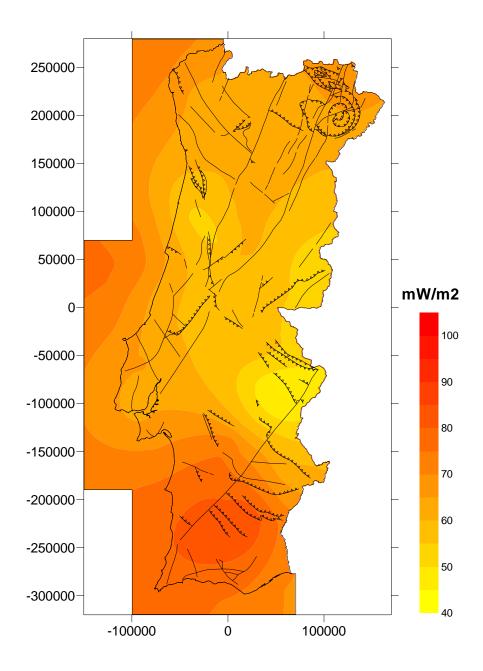




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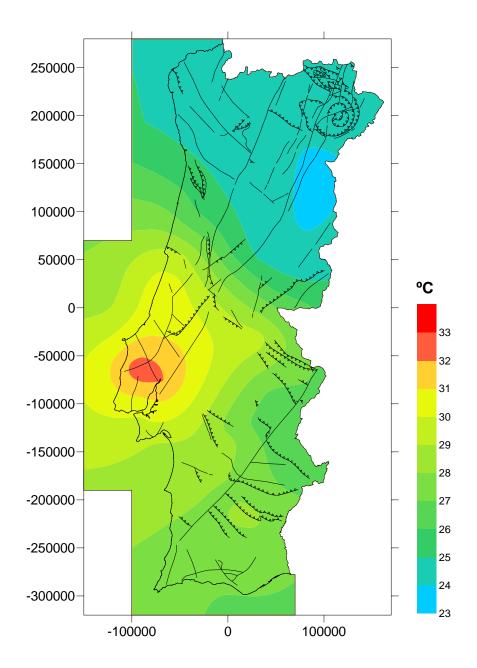
## CARTA DO GRADIENTE GEOTÉRMICO DE PORTUGAL 2014



Laboratório de Geologia e Minas



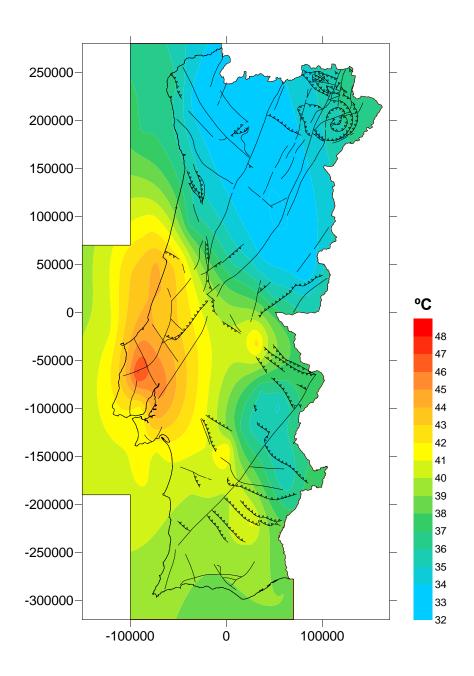
## CARTA DA DENSIDADE DE FLUXO DE CALOR À SUPERFÍCIE EM PORTUGAL 2014



Laboratório de Geologia e Minas



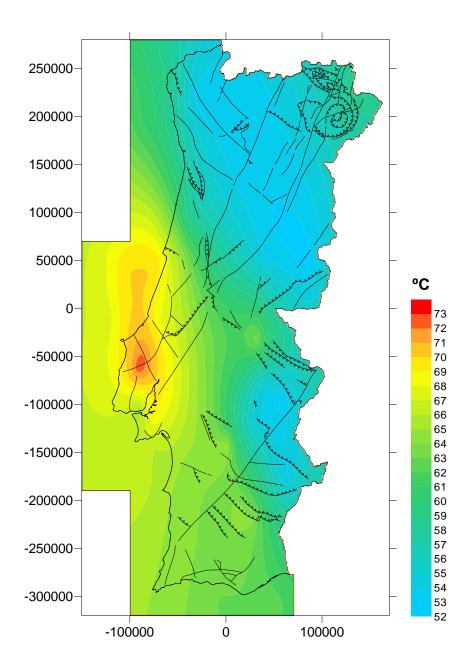
## CARTA DA TEMPERATURA A 500 m DE PROFUNDIDADE 2014



Laboratório de Geologia e Minas



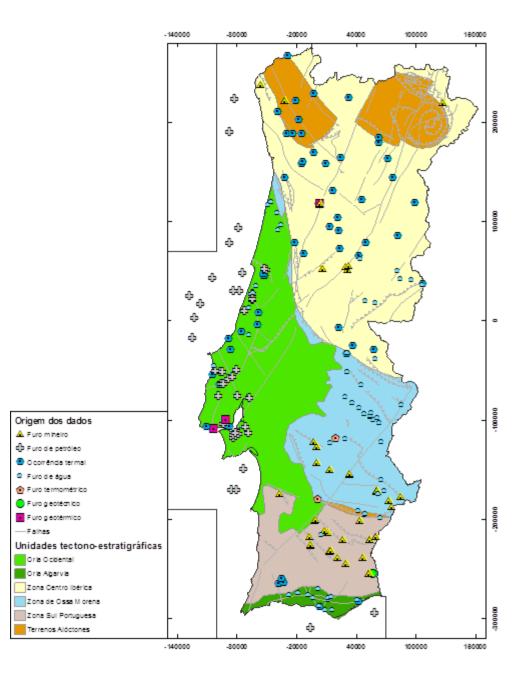
## CARTA DA TEMPERATURA A 1000 m DE PROFUNDIDADE 2014

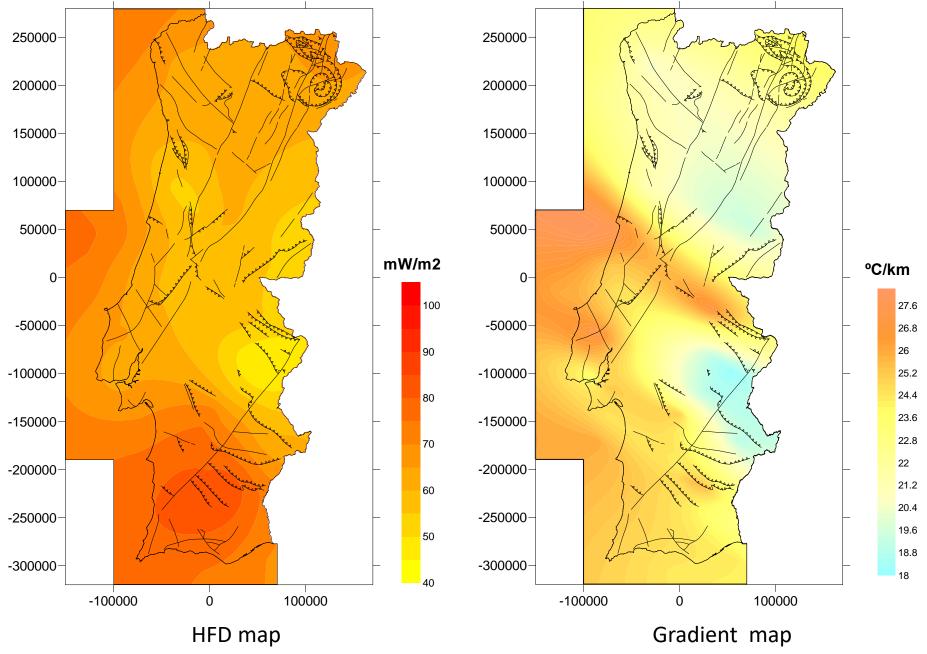


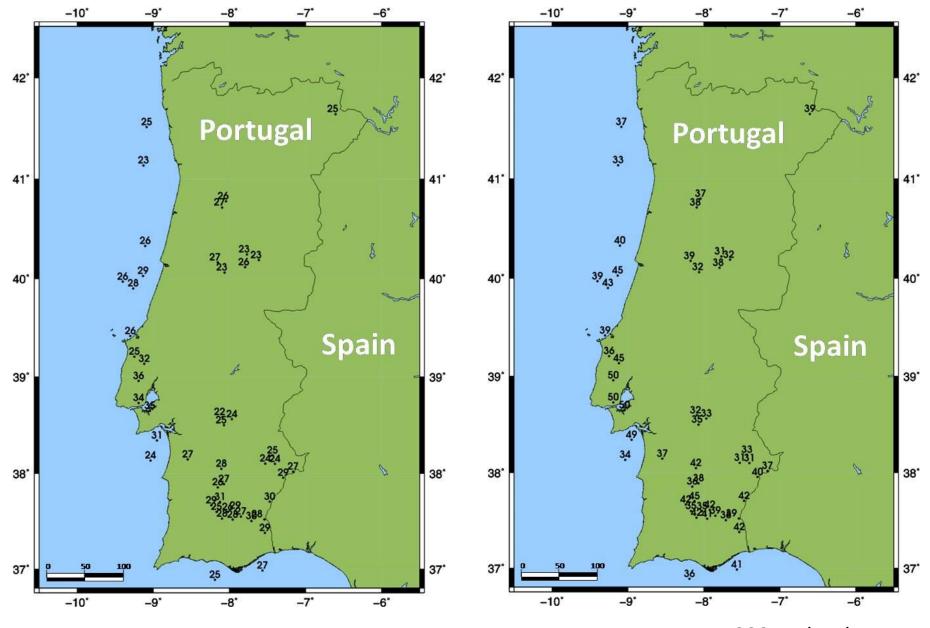
Laboratório de Geologia e Minas



## CARTA DA TEMPERATURA A 2000 m DE PROFUNDIDADE 2014

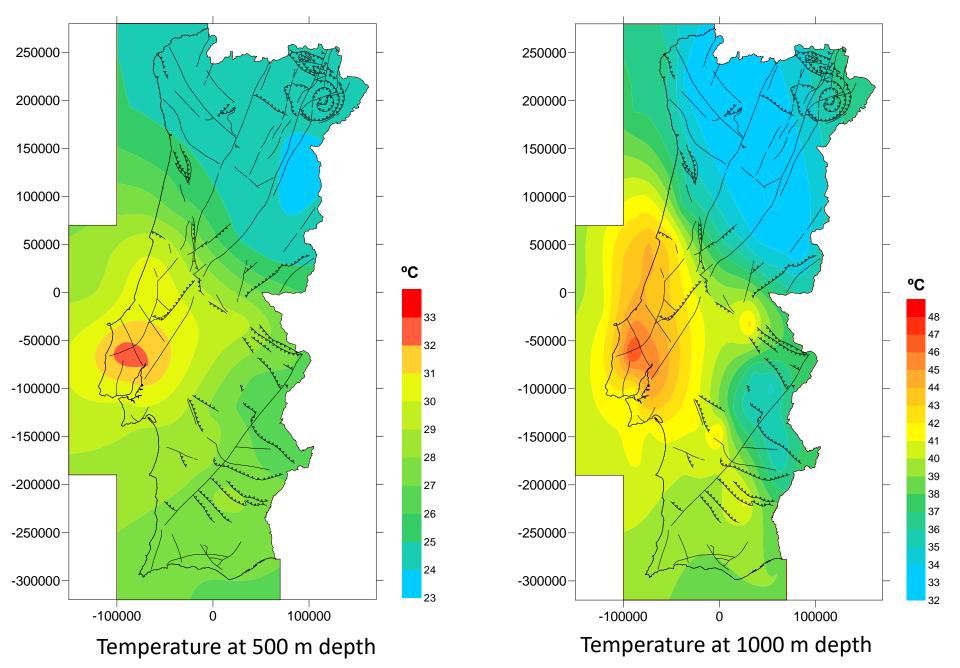


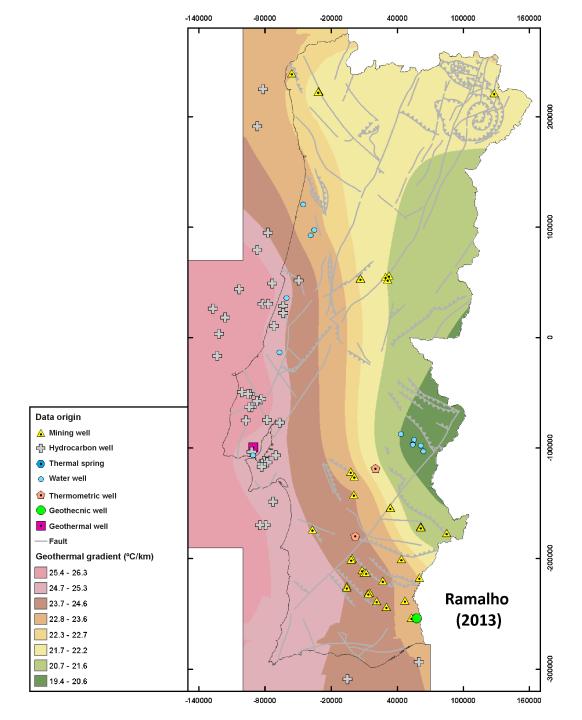




Temperature at 500 m depth

Temperature at 1000 m depth

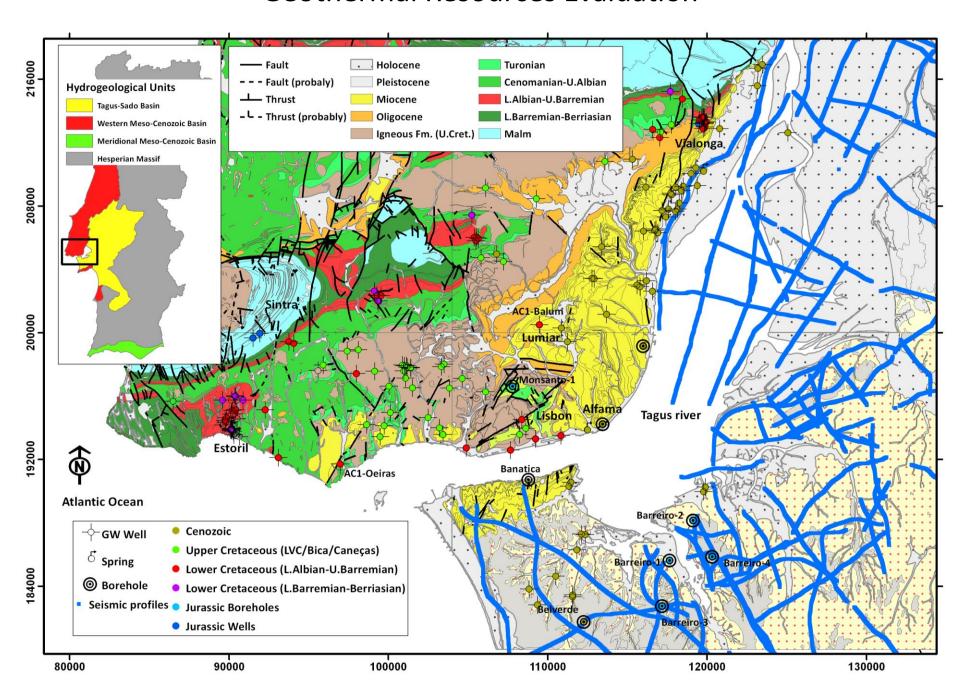


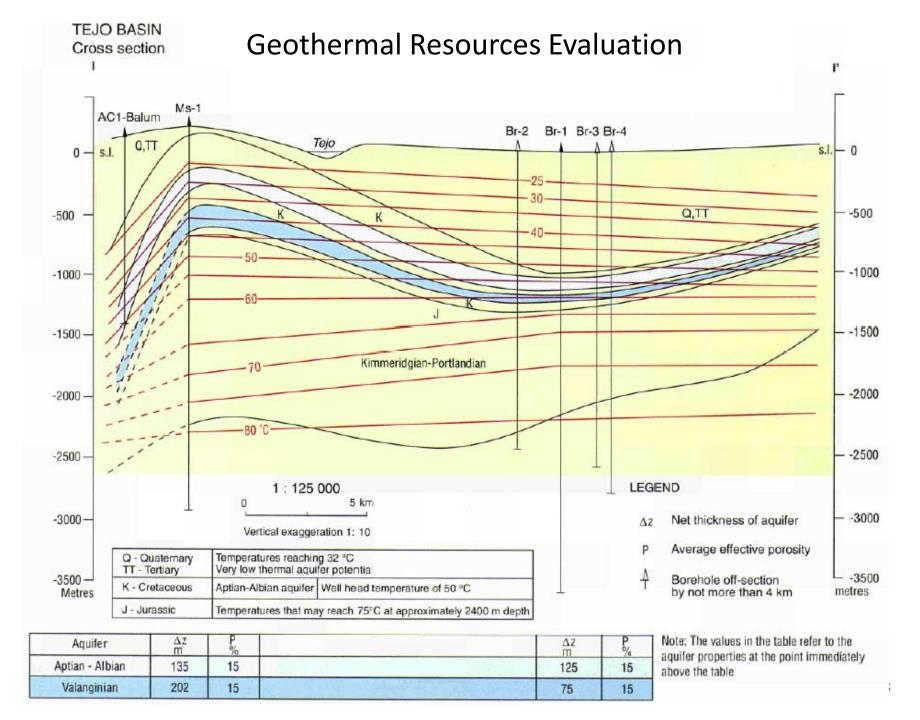


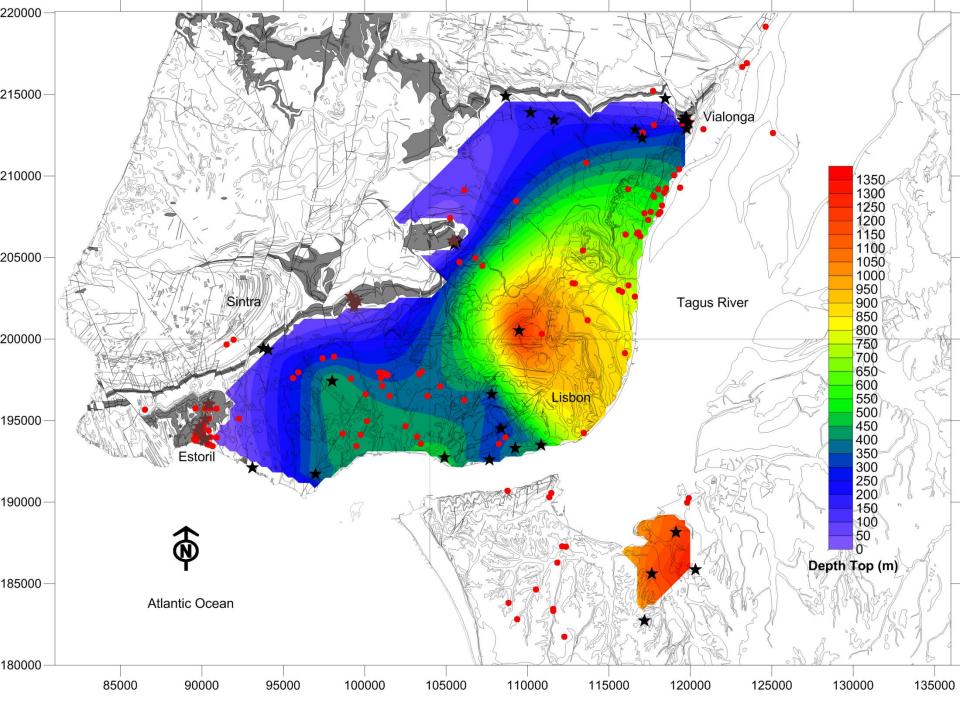
Geothermal Resources Evaluation

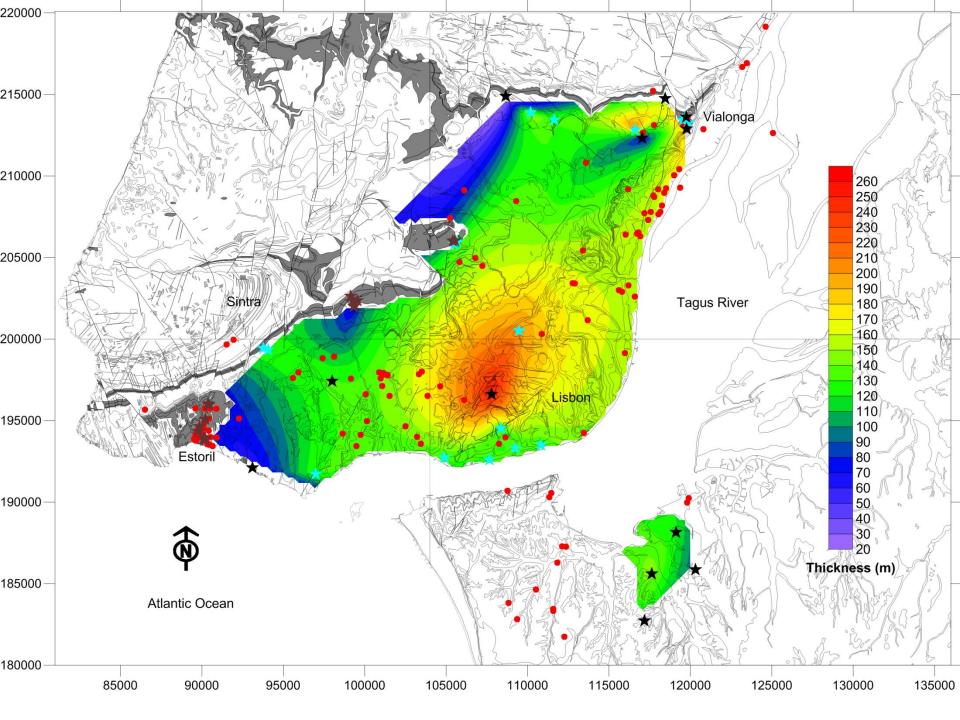
Lisbon region

#### **Geothermal Resources Evaluation**









### Heat in place

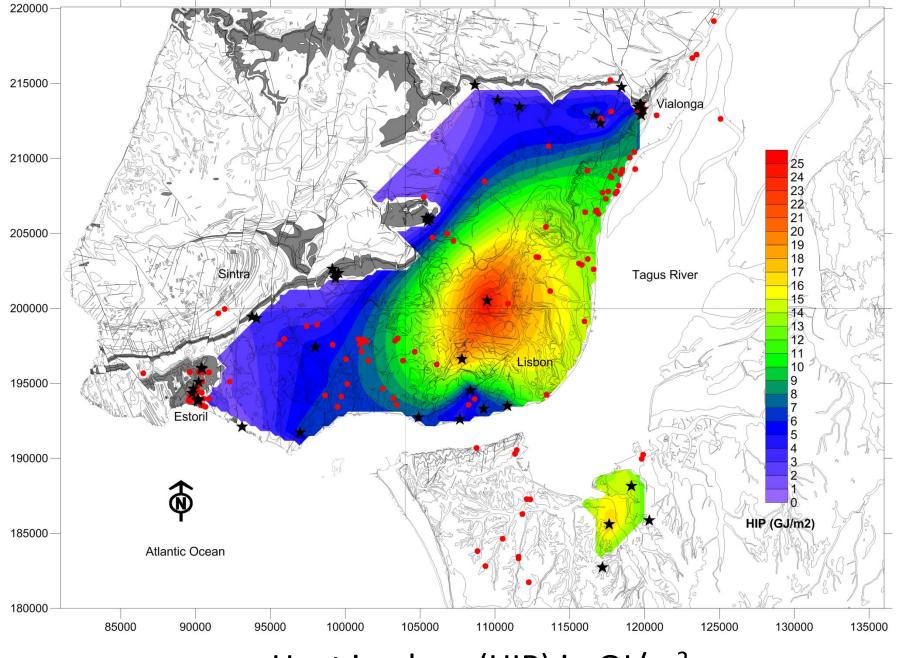
$$HIP = (\gamma_w \cdot \phi + \gamma_r \cdot (1 - \phi)) \cdot A \cdot h \cdot (T_r - T_0)$$

 $\gamma_w$  4180 J/kg·°C and  $\gamma_r$  value of 3740 kJ/m³·°C

#### Extractable Heat

$$EH = HIP \cdot R_g$$

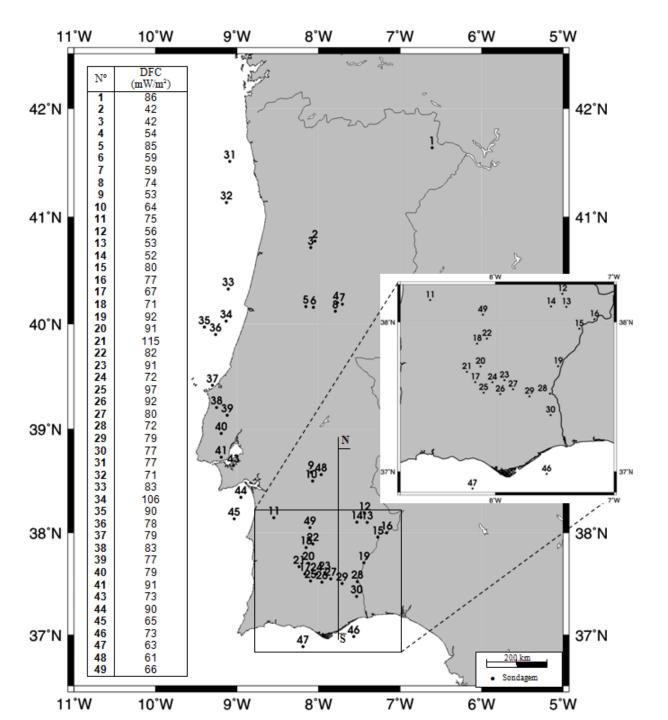
R<sub>g</sub> can vary between 0.05 to 0.1

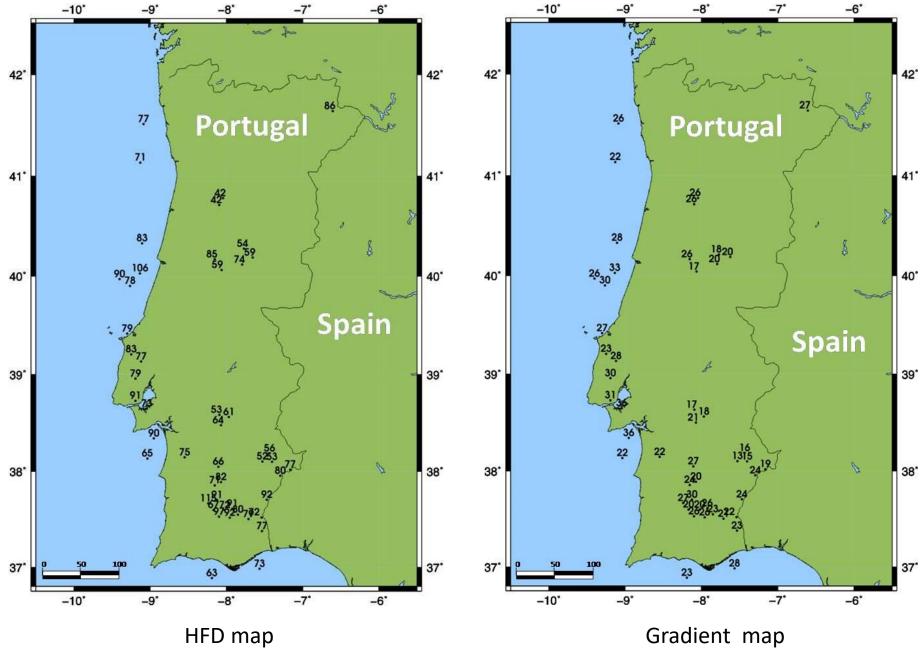


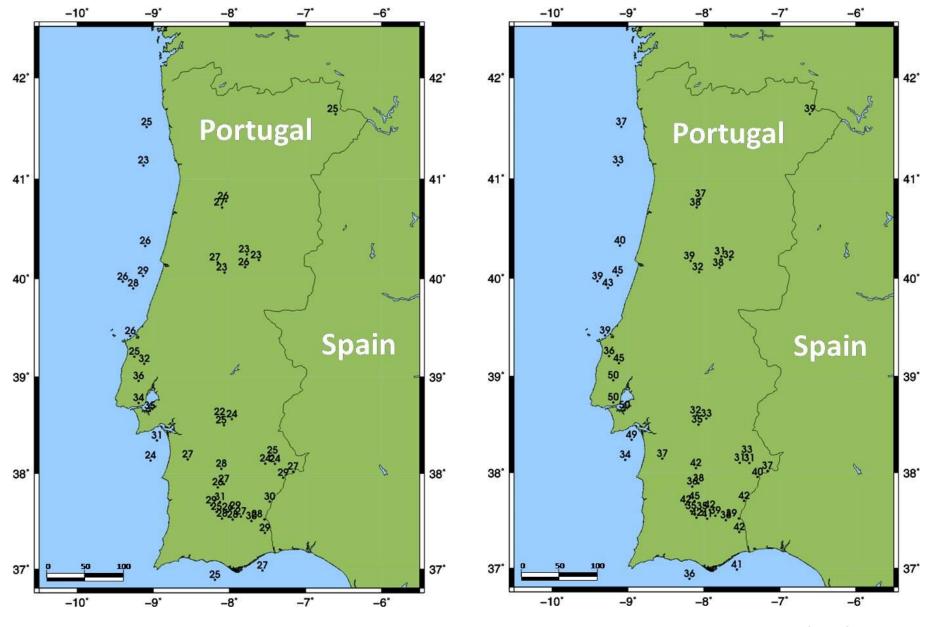
Heat in place (HIP) in GJ/m<sup>2</sup>

#### **Conclusions**

- The Heat in Place in the Lisbon area is about 3.8 GJ/m<sup>2</sup>.
- Assuming a recovery factor of 0.15, an Extractable Heat of 0.6 GJ/m<sup>2</sup> was obtained.
- This value is significantly lower than other deep sedimentary formations already exploited for district heating, such as the Paris Basin (7 GJ/m²) or the Upper Rhine Graben (15-30 GJ/m²).





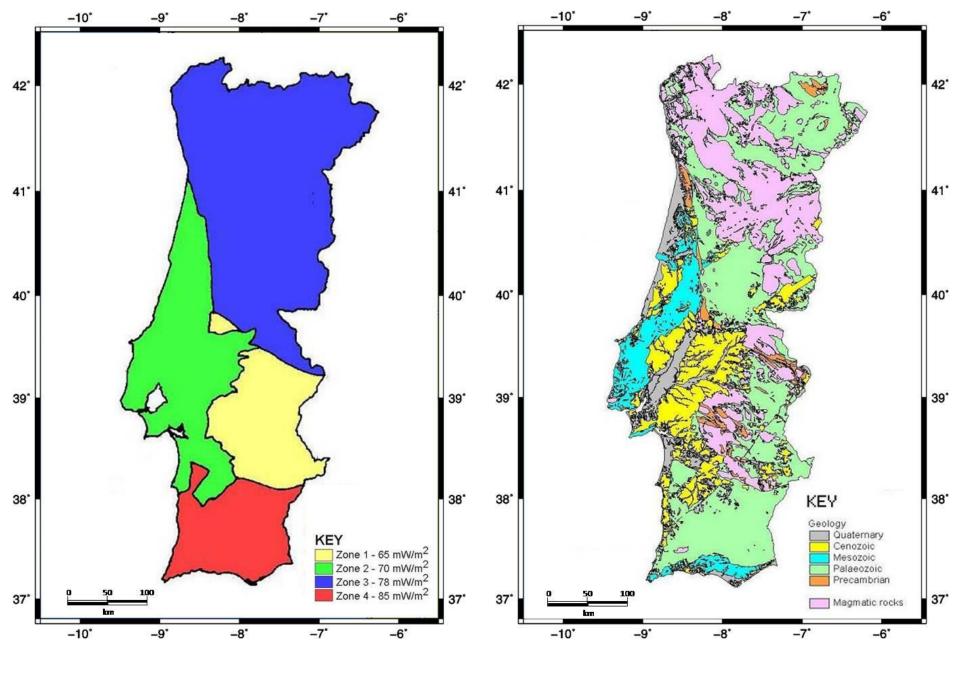


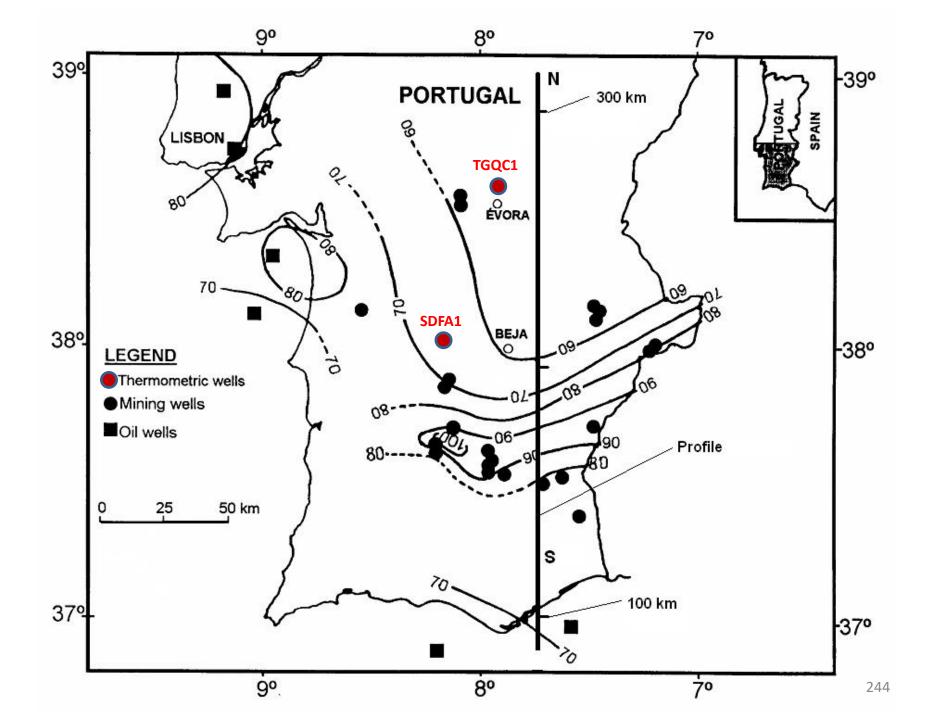
Temperature at 500 m depth

Temperature at 1000 m depth

## So, what can we do with this information?

- Study the thermal regime of the crust and mantle
- Use heat flux and geothermometers
- Construct maps of geothermal energy potential
- Study climatic change
- Study rheology of the crust and mantle





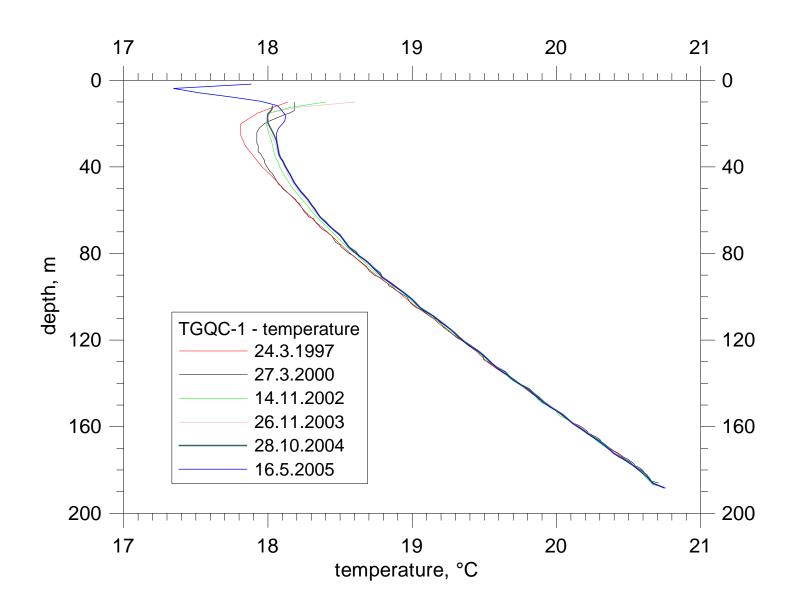


## Geothermal Climate Change Observatory in the TGQC-1 well

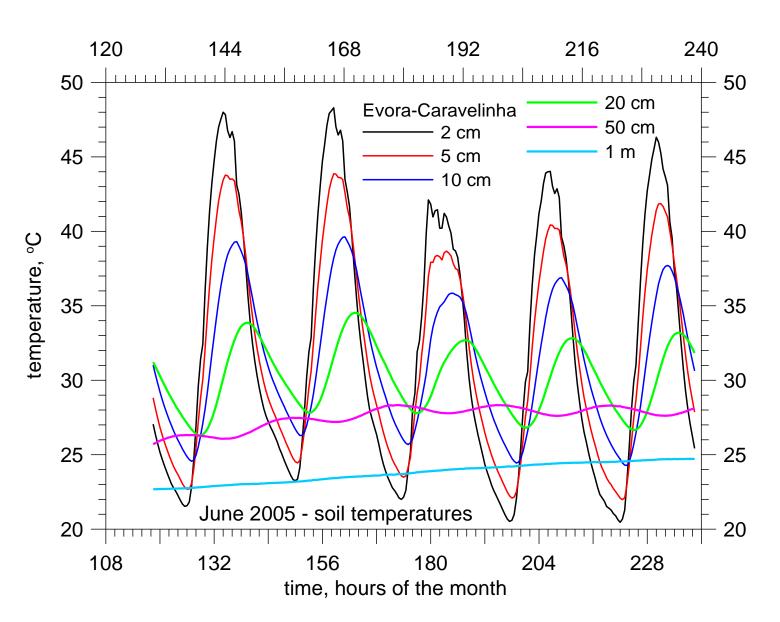
### Depth (m) of sensors in the borehole:

0.02 0.05 0.10 0.20 0.50 1.0 2.0 5.0 10.0 20.0 30.0 e 40.0

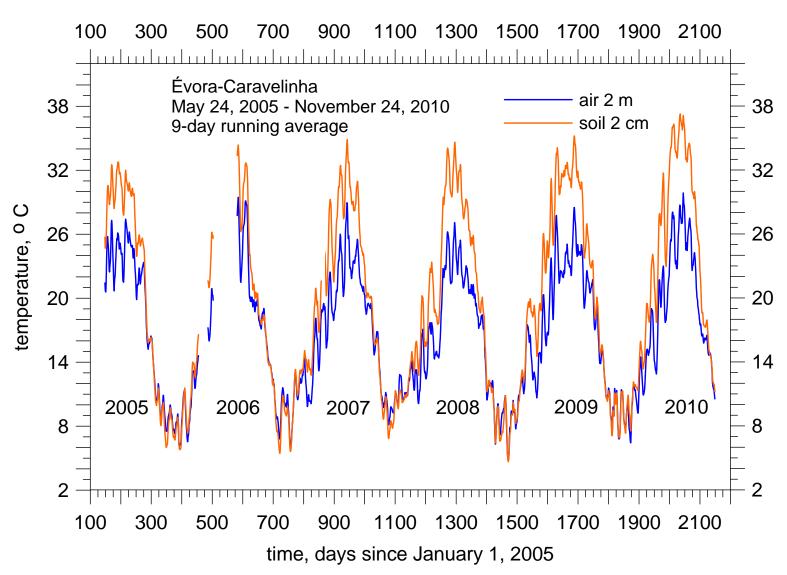
#### Temperature logs for different years



#### Temperature time series I



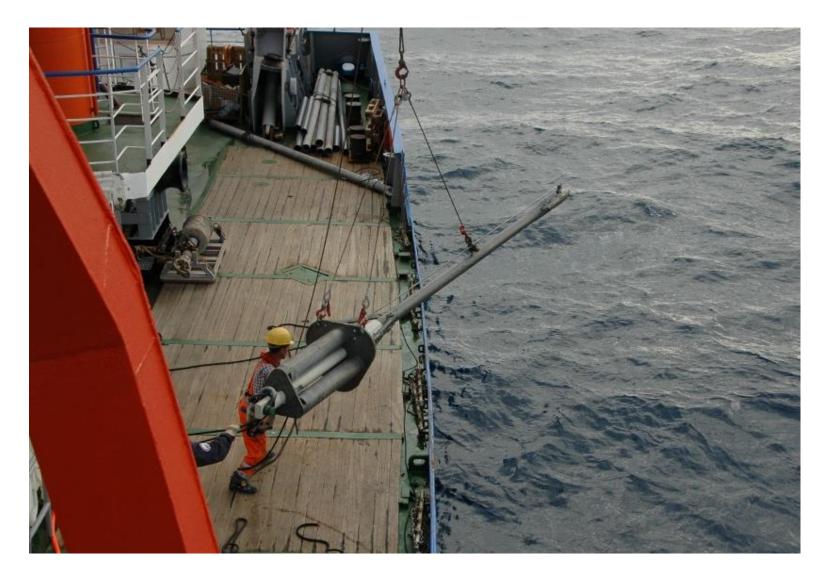
#### Temperature time series II

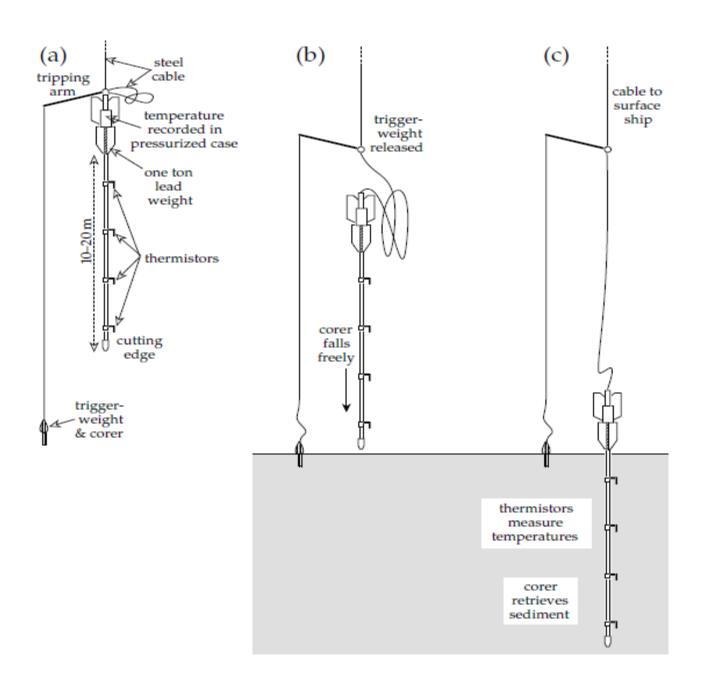


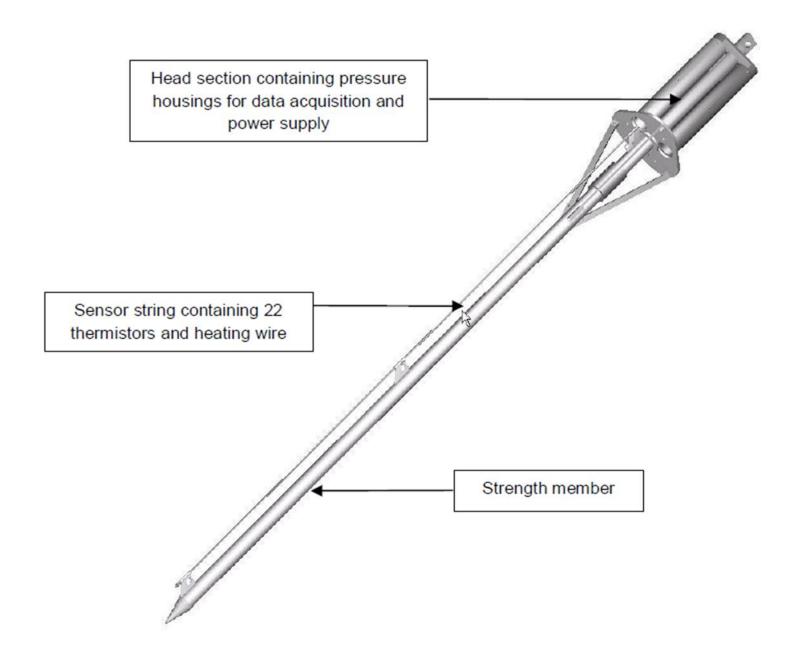
#### **Conclusions**

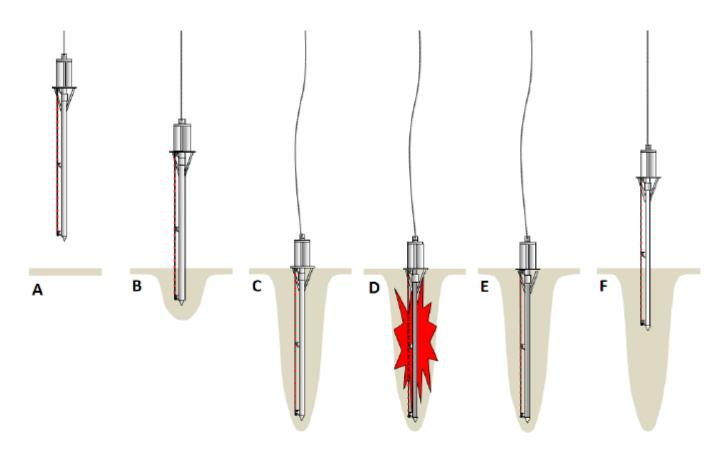
- Temperature logs can be a useful tool to infer the past climate change.
- The GST reconstruction for the TGQC-1 well indicates a warming trend.
- The warming trend appears to accelerate in the last 20 to 25 years.
- The installation of a geothermal climate change laboratory in the TGQC-1 well hopefully will allow understanding airground coupling and so constrain future GST inversions.

# VII Heat Flow Density in the Sea









A: lowering to seabed

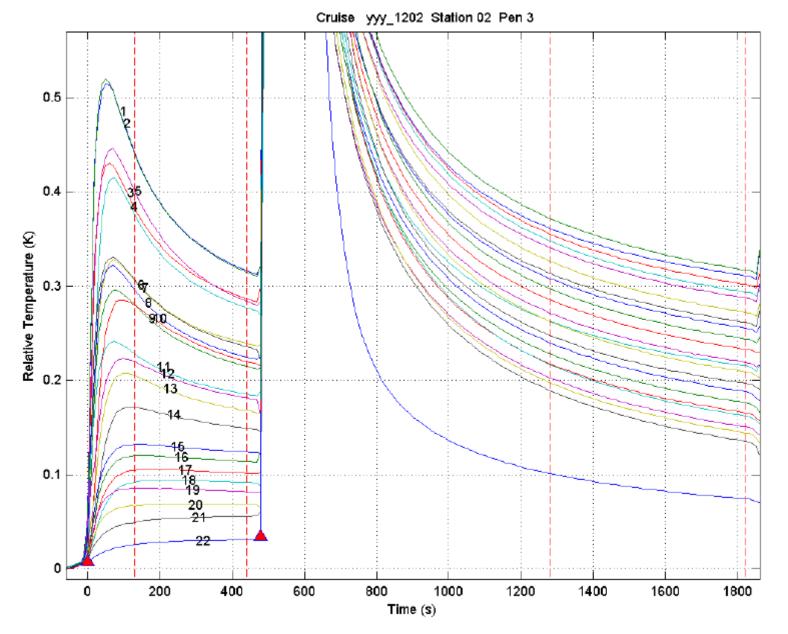
B: penetrating into seabed

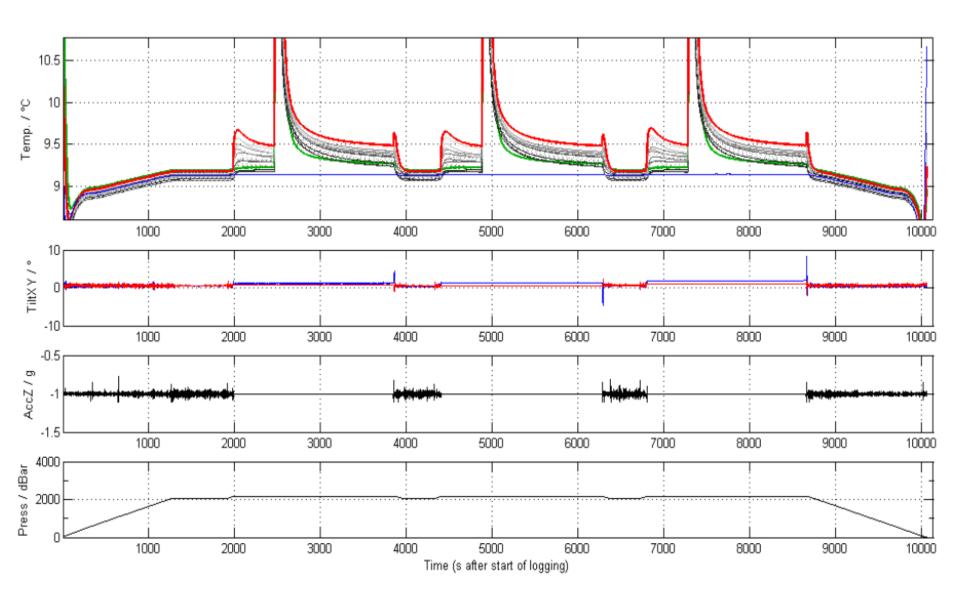
C: measuring thermal decay of frictional heat (approx.. 7-12 minutes)

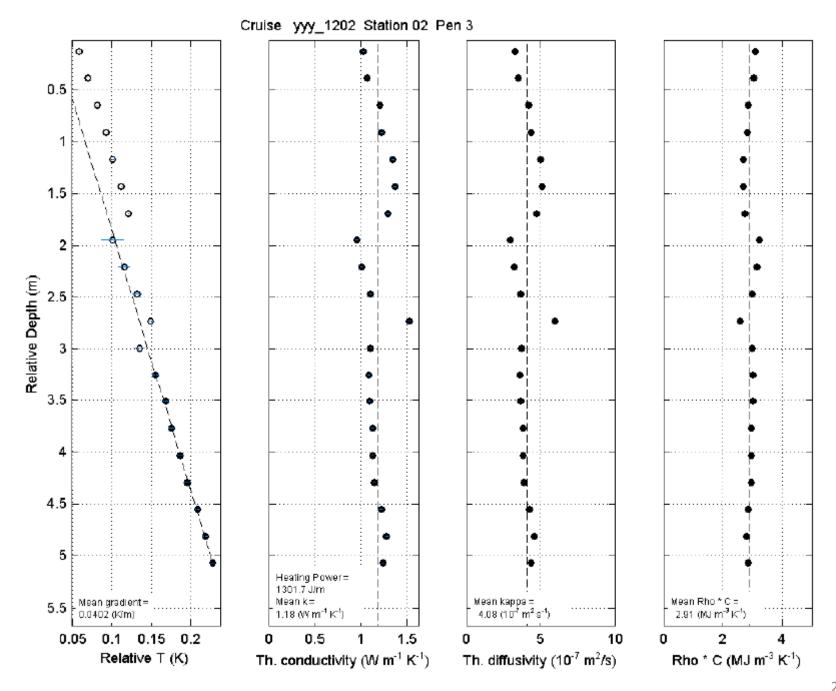
D: heat pulse (approx.. 20 seconds)

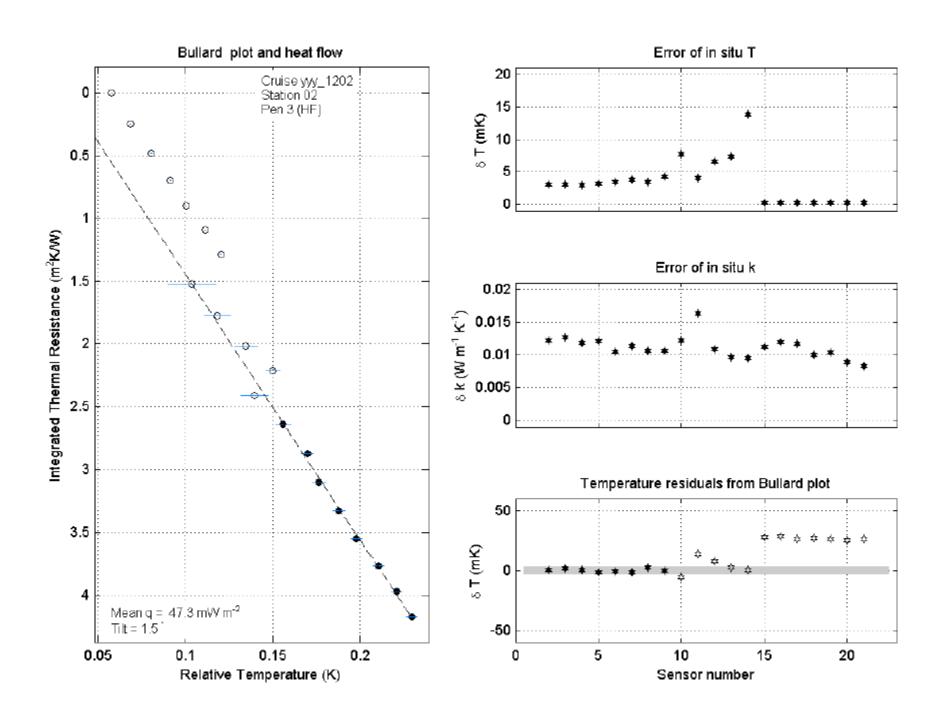
E: measuring thermal decay of heat pulse (approx.. 15-20 minutes)

F: pullout and retrieve to surface





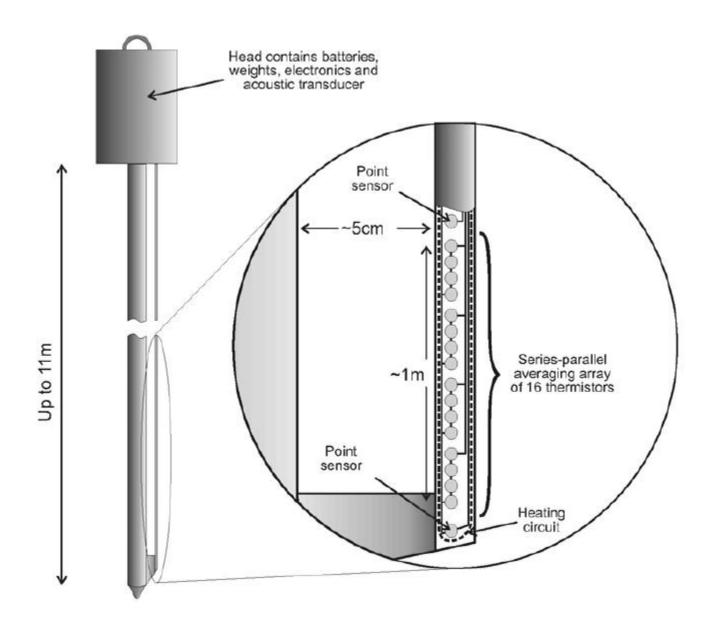




The approach that was talking here was to identify areas that show good prospects for high heat flow and then to use the heat flow probe to measure the temperatura and the seabed thermal gradient

La localización de las estaciones se definirá durante el crucero dependiendo de la información batimétrica y de cobertura de sedimentos que se obtenga mediante Multibeam, ecosonda y TOPAS.

Respecto a la localización de las estaciones, se especifica que las estaciones donde se realizarán las mediciones de flujo de calor terrestre con la sonda "FIELAX GmbH Heat Flow Probe" se seleccionarán con base en los datos del perfilador TOPAS, ya que es indispensable contar con una cubierta de sedimentos de espesor mayor a 10 m para que la sonda penetre y realice la medición del gradiente de temperatura.



A Lister "violin bow" heat flow probe