



1 The Impact of Compositional Data in Environmental 2 Risk Assessment through Information Theory

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12 **Abstract.** A water system impacted by mining activities was assessed to determine the extent of contamination, in the Trímpachos River mining system, in Spain. This system is in the Iberian Pyritic Belt, a metallogenic province in the southwest region of the Iberian Peninsula. Related pollution has been studied by multiple authors in recent decades. However, a pollution geochemical signature is not yet defined, even if, a few elements such as Cd, Cr, Cu, Fe, Hg, Mn, Pb, and Zn reach critical values, much above legislation for surface waters. Mercury is responsible for the highest level of hazard and therefore is central to defining water pollution signatures associated with acid drainage. Water samples were collected at the surface level of the streams, acidified with nitric solution, and stored in dark glass (only for Hg) and polyethylene containers at 4°C. Samples were digested with nitric and hydrochloric solutions in a high-pressure microwave unit and analyzed in ICP-OES for the majority of metals. Hg was directly analyzed in a mercury analyzer (NIC MA-3000). Since the chemical element concentration is compositional, an analysis was conducted to quantify how the uncertainty of the states of a to-be-predicted variable (mercury) is influenced by using both raw and centered log-ratio transformation (CLR) data. For that purpose, a methodology based on information theory (IT) and implemented through a Bayesian approach was used to about the obtained results, the normalized entropy decreased from 43% (raw data) to 33% (compositional data), and a Contingency Table Fit of 21% (raw data) was obtained compared to 71% (compositional data).

34
35 **Keywords:** Iberian Pyritic Belt, Pollution geochemical signature, Compositional
36 data, Information Theory.

37 1 Introduction

38 The compositional nature of geochemical data and other geo- and environmental sciences has been studied since the 1980s, when J. Aitchison started delving into the development of *compositional data analysis* (CoDa), introducing what is now known as

41 the *log-ratio approach* [1, 2, 3]. Specifically, compositional data is a representation of
 42 the parts of a whole, where each of its positive components (D) belonging to a random
 43 vector (Z) carries only relative information [4].

44 In this sense, it is widely recognized that the analysis and interpretation of regionalized
 45 compositions treated as raw data, although still commonly practiced, can easily
 46 lead to spurious correlations [5, 6]. Hence, the adoption of log-ratio transformation
 47 stands endorsed in the realm of environmental sciences, notably within the domains of
 48 geology and geochemistry [7, 8, 9]. One particularity of these analysis domains, such
 49 as contaminant flow control, is that in the field of uncertainty and probabilistic risk
 50 analysis, we can observe that both aleatory or stochastic in-situ uncertainty and epis-
 51 temic or subjective in-situ uncertainty are entirely dependent on the quantity and quality
 52 of the available data [10].

53 Therefore, the present study introduces a novel methodology based on the Infor-
 54 mation Theory (IT) introduced by Claude Shannon [11]. The proposed approach rec-
 55ognizes the inherent significance of information theory as a differential tool for uncer-
 56tainty interpretation, thereby tapping into its potential for informed decision-making a
 57 pollution geochemical signature is not yet defined and conducting probabilistic risk
 58 analysis. In this context, it also addresses the need to establish a well-informed pollution
 59 geochemical signature. For these purposes, a Bayesian machine learning (BayesianML)
 60 framework was developed to systematically assess regionalized compositions treated
 61 as raw data and establish a comparison with transformed variables using the centered
 62 log-ratio transformation (clr).

63 2 Materials and Methods

64 2.1 Data collection

65 The contamination levels of the Trimpancho River mining system in Spain, located in
 66 the metal-rich Iberian pyritic belt, were assessed. Water samples were collected from
 67 surface streams and tested for various elements including Al, Ca, Co, Cr, Cu, Fe, K,
 68 Mg, Mn, Na, Ni, Pb, Zn, Sulfate, Phosphate, Nitrate, and Hg. For the analysis of the
 69 metallic elements, samples were acidified using a nitric solution, stored in polyethylene
 70 containers at 4°C, and processed using a high-pressure microwave unit with nitric and
 71 hydrochloric solutions. Analysis was conducted using ICP-OES. Mercury was deter-
 72 mined in refrigerated samples stored in dark glass containers, using a mercury analyzer
 73 (NIC MA-3000) based on thermal decomposition, gold amalgamation, and cold vapor
 74 atomic absorption spectroscopy detection. Nitrates, phosphates, and sulfates were ana-
 75 lyzed in non-acidified samples, nitrates by a portable photometer, and phosphates and
 76 sulfates by UV-Vis spectrophotometry.

77 2.2 Data transformation

78 The initial step of the analysis entails the transformation of the raw data to real space
 79 (clr-coefficients) based on the centred log-ratio transformation (clr) [3]. To that end,
 80 the compositional data transformation was performed using CoDaPack v2 software [12]

$$81 \quad y = \text{clr}(x) = \left[\ln \frac{x}{g_D(x)} \right] = \left[\ln \frac{x_1}{g_D(x)}, \ln \frac{x_2}{g_D(x)}, \dots, \ln \frac{x_D}{g_D(x)} \right] \quad (1)$$

82 where $\mathbf{y} \in \mathbb{R}^{D-1}$ and $g_D(\mathbf{x})$ is the geometric mean of the parts involved.

83 2.3 Entropy and Mutual Information

84 The objective of knowledge modeling and reasoning with BayesianML is to anticipate
 85 and understand the consequences of uncertainty, whether positive or negative. For that,
 86 one can express the quantification of normalized uncertainty $H_n(x)$ associated with the
 87 probability distribution of a variable X or a set of variables G as [11, 13]:

$$H_n(X) = \frac{H(X)}{\log_2(\phi_X)} = \frac{-\sum_{x \in X} p(x_i) \log_2(p(x_i))}{\log_2(\phi_X)} \quad (2)$$

where x_1, \dots, x_n represent the potential outcomes of X, each occurring with a corresponding probability of $p(x_1), \dots, p(x_n)$, while ϕ_x represents the total number of states of a variable X. In addition to Shannon's entropy expression, another essential parameter in information theory is the mutual information (MI). The MI conceptually describes the interdependence between two variables, X and Y, by means of their information content. Mathematically, from an entropy perspective, MI can be expressed as:

$$95 \quad MI(X, Y) = H(X) - H(X|Y) \quad (3)$$

96 where $H(X)$ represents the marginal entropy, and $H(X|Y)$ the conditional entropy.

97 3 Results

98 3.1 Exploratory Analysis

99 In the first stage of analysis, a primary Bayesian model is created to evaluate the association rate of variables in terms of probabilistic relationships between nodes (Fig. 1).
100

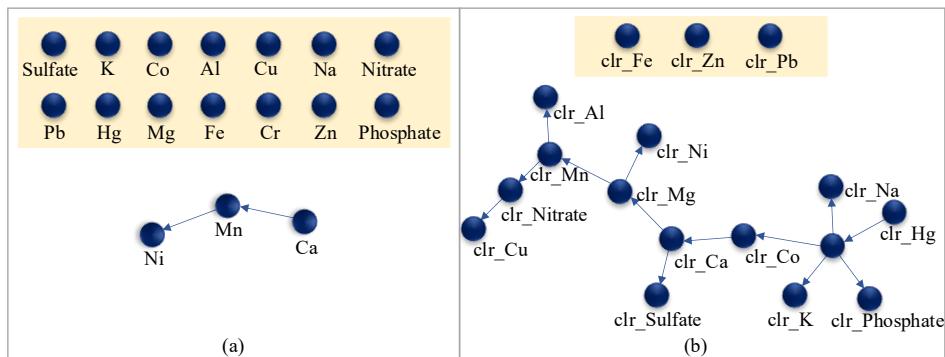


Fig. 1. A variation of the standard 1-bit error (a) $R = 1$, (b) $R = 1/3$, (c) $R = 1/2$.

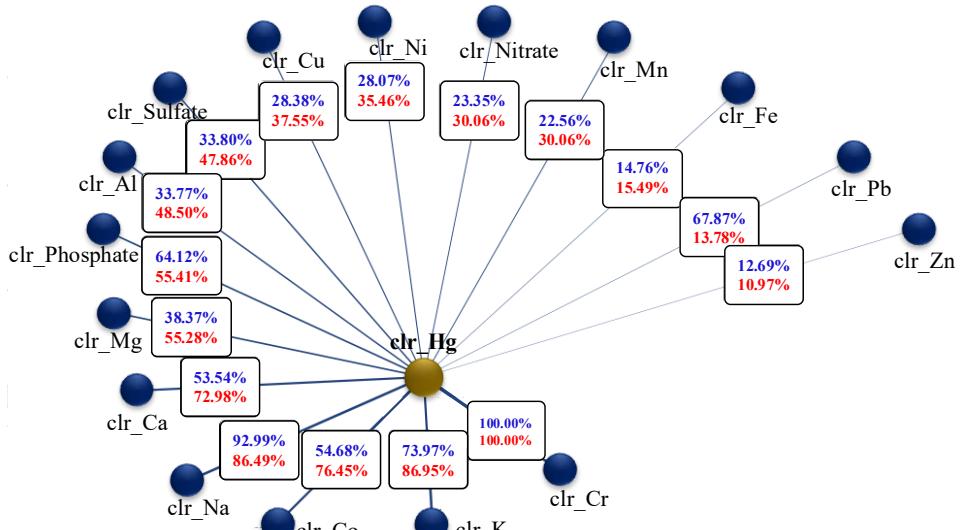
103 In Fig. 1, a significant improvement can be observed in uncovering potential relationships between variables when transformed compositional data is employed. Additionally, the uncertainty of the CoDa model was reduced from 43% (raw data) to 33%. Among these results, the most remarkable findings were seen in the Contingency Table Fit of the compositional data model, with a significant 41% improvement versus the raw data BayesianML network, from 21% to 70%.

109 **3.2 Supervised Analysis on Hg**

110 Fig. 2 shows the analysis of mutual information. The upper number in the box represents the information exchanged relative to the secondary node, while the red number refers to the main node. Otherwise, the symmetric measure of the information exchanged between each node and the target node is graphically represented by the thickness of the arc and its distance from the target node. This symmetric representation means that the amount of information, for example, supplied by node K about Hg is the same as the amount of information Hg supplied about node K. Thanks to this information analysis, it is possible to identify the predictive importance of variables such as Cr, K, Co, Na, and Co for understanding the state of Hg, considering the available data set in February of 2022. A new campaign was conducted in February 2023 and the new dataset will be reflected in the calibration model.

121

122 **Fig. 2.** Radial hierarchy layout from the highest-ranked node to the lowest-ranked information node.



123

124 **4 Discussion**

125 In mine exploration, environmental impacts, among others, the general focus, concerning
 126 geochemical signatures' definition is traditionally based on the uncertainty arising
 127 from sparse data and not on uncertainty arising from the model, even though the model
 128 is inferred, and its parameters estimated. The total Hg content of the Trímpanco River
 129 water in the present survey is the total Hg. The composition of Hg can be divided into
 130 different forms or pools, such as "dissolved" Hg, Hg associated with particulate and
 131 colloidal matter, volatile elemental Hg0, and labile (or reactive) Hg(II) [14]. The pre-
 132 dictive importance of variables such as Cr, K, and Co, for Mercury's fate interpretation,
 133 can be explained by the colloid particulate-bound in the Hg forms, which can have
 134 identical association with the signalized elements. Divalent mercury, readily soluble as
 135 HgCl₂, can finally explain the importance of Na, usually associated with Cl-, in the
 136 water column of Mediterranean rivers.

137 **5 Conclusions**

138 Considering the definition of future geochemical signatures, for the Trímpanco
 139 River's pollution characterization, the relationships between elements must be evalua-
 140 ted considering the Hg contents in its dissolved forms, in the water column, as well as
 141 the forms in which this element occurs in the deposited sediments, which represent the
 142 largest pool of this element in these polymetallic-sulfide mining areas, which enrich in
 143 cinnabar (HgS).

144 The collected samples were log-centred transformed after which a BayesianML
 145 analysis was carried out using the Information Theory fundaments for uncertainty and
 146 mutual information quantification. The findings revealed a significant increase in un-
 147 derstanding of the study area by exploring transformed analytical data. In addition,
 148 CoDa not only facilitated the identification of preferred associations but also provided
 149 a comprehensive framework for defining water pollution signatures. The authors be-
 150 lieve that this approach will provide valuable insights that will pave the way for more
 151 effective management and mitigation strategies in the future.

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